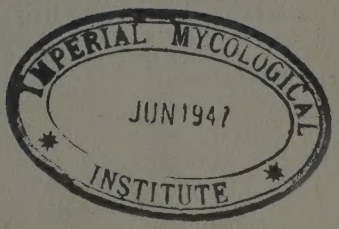
COMPTES RENDUS
(DOKLADY)
DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1944



VOLUME XLIV

№ 8

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
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Paraissant tous les dix jours

1944

VOLUME XLIV, № 8

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MATHEMATICS

THE FOURIER INTEGRAL AND FUNCTIONALLY INVARIANT SOLUTIONS OF THE WAVE EQUATION IN n-DIMENSIONAL SPACE

By V. G. GOGOLADZE

(Communicated by S. L. Soboleff, Member of the Academy, 12.II.1944)

V. I. Smirnoff, M. A. (1), and S. L. Soboleff, M. A. (2), have constructed an important class of functionally invariant solutions of the wave equation of the theory of elasticity

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=\frac{1}{a^2}\frac{\partial^2\Phi}{\partial t^2}; \qquad \frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}=\frac{1}{b^2}\frac{\partial^2\Psi}{\partial t^2} \tag{1}$$

in two-dimensional space. Our purpose is to construct this class of solutions of the wave equation in a space of many dimensions. The construction is based on the representation of functions by the Fourier integral. For the sake of simplicity we restrict ourselves to the case of two-dimensional space. The construction for a space of many dimensions can be carried out analogously.

Suppose that a force perpendicular to $y=0$ begins to act at the moment $t=0$ on the segment Ω.

We represent it by Fourier integral

$$Q=\frac{1}{4\pi^2}\int\int_{-\infty}^{+\infty}\int\int Q(\xi,\tau)\,e^{i[\alpha(x-\xi)+p(t-\tau)]}\,d\xi\,d\alpha\,dp\,d\tau \tag{2}$$

Let us write the solution of the equation (1) with the source of vibrations (2) in the form

$$\Phi=\frac{1}{4\pi^2\mu}\int_{-\infty}^{+\infty}\cdots\int\frac{Q(\xi,\tau)}{2\frac{p^2}{b^2}}\,e^{i\left[\alpha(x-\xi)+p(t-\tau)-y\sqrt{\frac{p^2}{a^2}-\alpha^2}\right]}\,d\xi\,d\alpha\,dp\,d\tau \tag{3}$$

$$\Psi=\frac{1}{4\pi^2\mu}\int_{-\infty}^{+\infty}\cdots\int\frac{Q(\xi,\tau)\,\alpha}{2\frac{p^2}{b^2}\sqrt{\frac{p^2}{a^2}-\alpha^2}}\,e^{i\left[\alpha(x-\xi)+p(t-\tau)-y\sqrt{\frac{p^2}{b^2}-\alpha^2}\right]}\,d\xi\,d\alpha\,dp\,d\tau \tag{4}$$

where the radicals $\sqrt{\frac{p^2}{a^2}-\alpha^2}$ and $\sqrt{\frac{p^2}{b^2}-\alpha^2}$ are positive for $p>0$ and $\frac{p^2}{b^2}-\alpha^2>0$.

Consider the case $y>0$ (the case $y<0$ may be discussed analogously) and let us study the integral (3) [the integral (4) is quite similar].

In the following we shall have to change the order of integration. This will be found upon immediate verification to be a lawful procedure in so far as we shall see that the obtained expression for Φ will satisfy the wave equations as well as all physical conditions of the problem.

The integral (3) is investigated by N. Zeilon's method ([3]) which has been applied by E. Hallen ([4]) to the problem of vibration of an elastic semi-space. We put (3) in the form

$$\Phi=\frac{i}{4\pi^2\mu}\frac{\partial}{\partial y}\int_{\Omega}d\xi\int_{-\infty}^{+\infty}d\tau\int_{-\infty}^{+\infty}d\alpha\int_{\Gamma}\frac{Q(\xi,\tau)\,e^{i|\alpha|\left[\frac{\alpha}{|\alpha|}(x-\xi)+\zeta(t-\tau)-y\sqrt{\frac{\zeta^2}{a^2}-1}\right]}}{2a^2\frac{\zeta^2}{b^2}\sqrt{\frac{\zeta^2}{b^2}-1}}d\zeta \tag{5}$$

where $p=|\alpha|\zeta$, Γ is a real path of integration $(-\infty,\ \infty)$ lying in the first sheet of the Riemann surface with the branch points $\pm a$. For $\zeta>a$, $\sqrt{\frac{\zeta^2}{a^2}-1}>0$.

By changing the variables

$$H=\frac{\alpha}{|\alpha|}(x-\xi)+\zeta(t-\tau)-y\sqrt{\frac{\zeta^2}{a^2}-1} \tag{6}$$

we obtain

$$\Phi=\frac{ia}{4\pi^2\mu}\frac{\partial}{\partial y}\int_{\Omega}d\xi\int_{-\infty}^{+\infty}d\tau\int_{-\infty}^{+\infty}d\alpha\int_{\Gamma}\frac{Q(\xi,\tau)\,e^{i|\alpha|H}dH}{2a^2\frac{\zeta^2}{b^2}\sqrt{\left[H-\frac{\alpha}{|\alpha|}(x-\xi)\right]^2+y^2-a^2(t-\tau)^2}} \tag{7}$$

The meaning of the integral with respect to α is in accordance with Zeilon-Hadamard ([5]).

We further have

$$\Phi=\frac{ia}{4\pi^2\mu}\frac{\partial}{\partial y}\int_{\Omega}d\xi\int_{-\infty}^{+\infty}d\tau\int_{\Gamma_1'}\frac{Q(\xi,\tau)\,f_1\,dH}{2\frac{\zeta'^2}{b^2}\sqrt{[H+(x-\xi)]^2+y^2-a^2(t-\tau)^2}}+$$

$$+\frac{ia}{4\pi^2\mu}\frac{\partial}{\partial y}\int_{\Omega}d\xi\int_{-\infty}^{+\infty}d\tau\int_{\Gamma_1''}\frac{Q(\xi,\tau)\,f_2\,dH}{2\frac{\zeta''^2}{b^2}\sqrt{[H-(x-\xi)]^2+y^2-a^2(t-\tau)^2}} \tag{8}$$

where Γ_1', Γ_1'' are real paths in the H-plane for $H>0$, $\arg H=0$. When $H<0$, $\arg H=\pi$; ζ' and ζ'' correspond to $\alpha/|\alpha|=-1$ and $\alpha/|\alpha|=+1$; the point $H=0$ is to be rounded in the upper semi-plane (to guarantee H to be real, the contour Γ in the ζ-plane in (5) must be somewhat deformed);

$$f_1=\int_{-\infty}^{-\delta}\frac{e^{-i\alpha H}}{\alpha^2}d\alpha+\int_{-\delta}^{0}\frac{e^{-i\alpha H}-1+i\alpha H}{\alpha^2}d\alpha$$

$$f_2=\int_{0}^{\delta}\frac{e^{i\alpha H}-1-i\alpha H}{\alpha^2}d\alpha+\int_{\delta}^{\infty}\frac{e^{i\alpha H}}{\alpha^2}d\alpha \tag{9}$$

where δ is an arbitrary positive number. We have

$$f_1=f_2=\frac{1}{\delta}+iH\left[1+\frac{\pi}{2}i-0.5772\ldots-\log\delta\right]-iH\log H \tag{10}$$

where $C=0.5772\ldots$ is Euler's constant.

The circuit around the point $H=0$ gives the increment $-2\pi i$ for $\log H$. The branch points in the ζ-plane corresponding to $H=0$ are determined, in view of (6), by the equations

$$\zeta(t-\tau)-y\sqrt{\frac{\zeta^2}{a^2}-1}-(x-\xi)=0 \tag{11}$$

$$\zeta(t-\tau)-y\sqrt{\frac{\zeta^2}{a^2}-1}+(x-\xi)=0 \tag{12}$$

Assume $r=\sqrt{(x-\xi)^2+y^2}<a(t-\tau)$. In this case the roots of the equations (11), (12) lie in the lower semi-plane ζ and are symmetric with respect to the imaginary axis:

$$\zeta_1=a\frac{a(t-\tau)-iy\sqrt{a^2(t-\tau)^2-r^2}}{a^2(t-\tau)^2-y^2}$$

$$\zeta_2=a\frac{-a(t-\tau)(x-\xi)-iy\sqrt{a^2(t-\tau)^2-r^2}}{a^2(t-\tau)^2-y^2} \tag{13}$$

Integration with respect to H is equivalent to integration with respect to ζ along two loops around the branch points ζ_1 and ζ_2 with end points at infinity. Let these loops $(\infty\,\zeta_1)$, $(\infty\,\zeta_2)$ be situated symmetrically with respect to the imaginary axis (4). We shall have

$$\Phi=\frac{i}{2\pi^2\mu}\frac{\partial}{\partial y}\int_{\Omega}d\xi\int_{-\infty}^{+\infty}d\tau\int_{\infty}^{\zeta_1}Q(\xi,\tau)\frac{\zeta(t-\tau)-y\sqrt{\frac{\zeta^2}{a^2}-1}-(x-\xi)}{2\frac{\zeta^2}{b^2}\sqrt{\frac{\zeta^2}{a^2}-1}}d\zeta+$$

$$+\frac{i}{2\pi^2\mu}\frac{\partial}{\partial y}\int_{\Omega}d\xi\int_{-\infty}^{+\infty}d\tau\int_{\infty}^{\zeta_2}Q(\xi,\tau)\frac{\zeta(t-\tau)-y\sqrt{\frac{\zeta^2}{a^2}-1}+(x-\xi)}{2\frac{\zeta^2}{b^2}\sqrt{\frac{\zeta^2}{a^2}-1}}d\zeta \tag{14}$$

where $\sqrt{\frac{\zeta^2}{a^2}-1}$ and ζ assume the complex values on the paths $(\infty\,\zeta_1)$ and $(\infty\,\zeta_2)$ differing only by the sign of their real parts. Hence (14) can be put in a more compact form:

$$\Phi=\mathrm{Re}\,\frac{i}{\pi\mu}\frac{\partial}{\partial y}\int_{\Omega}d\xi\int_{-\infty}^{+\infty}d\tau\int_{\infty}^{\zeta_1}Q(\xi,\tau)\frac{\zeta(t-\tau)-y\sqrt{\frac{\zeta^2}{a^2}-1}-(x-\xi)}{2\frac{\zeta^2}{b^2}\sqrt{\frac{\zeta^2}{a^2}-1}} \tag{15}$$

where, in virtue of (11), ζ_1 is complex for $r<a(t-\tau)$, $y>0$, and $\Phi\neq 0$. For $r>a(t-\tau)$, ζ_1 is real and exceeds a in absolute magnitude. In this case (15) gives $\Phi=0$. The elementary front of the wave is determined by the equation

$$(x-\xi)^2+y^2=a^2(t-\tau)^2 \tag{16}$$

Since the force Q is distributed on the segment Ω and begins to act at $t=0$, we obtain from (15) the following expression for the potential Φ

$$\Phi=\mathrm{Re}\,\frac{i}{\pi\mu}\frac{\partial}{\partial y}\int_{\Omega}d\xi\int_{0}^{t-\frac{r}{a}}d\tau\int_{\infty}^{\zeta_1}Q(\xi,\tau)\frac{\zeta(t-\tau)-y\sqrt{\frac{\zeta^2}{a^2}-1}-(x-\xi)}{2\frac{\zeta^2}{b^2}\sqrt{\frac{\zeta^2}{a^2}-1}}d\zeta \tag{17}$$

It is easy to show that the solution (17) just obtained satisfies the equation (1), as well as the conditions at the source and at infinity. We thus obtain a functionally invariant solution of the wave equation. The potential Ψ (4) can be studied similarly. Such a representation of an integral of the wave equation is valuable for the solution of problems of the dynamical theory of elasticity ([6–9]).

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Received
12. II. 1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 8

HYDRODYNAMICS

ON THE PROBLEM OF TURBULENCE

By L. LANDAU

(Communicated by P. L. Kapitza, Member of the Academy, 26.I.1944)

Although the turbulent motion has been extensively discussed in literature from different points of view, the very essence of this phenomenon is still lacking sufficient clearness. To the author's opinion, the problem may appear in a new light if the process of initiation of turbulence is examined thoroughly.

In the case of incompressible fluid the unsteadiness of the laminar motion is known to be determined as follows. Upon the principal motion with a velocity distribution $\vec{v}_0(x, y, z)$ there is superimposed a small disturbance $\vec{v}_1(x, y, z, t)$; the substitution of $\vec{v} = \vec{v}_0 + \vec{v}_1$ in the equation of motion of a viscous fluid and the neglect of terms of the second order of smallness lead to a linear differential equation for the perturbation $\vec{v}_1$. Further, $\vec{v}_1$ is sought in the form

$$\vec{v}_1 = A(t)\vec{f}(x, y, z) \qquad (1)$$

where the time function $A(t)$ may be represented as

$$A(t) = \text{const} \cdot e^{-i\Omega t} \qquad (2)$$

The problem of determining the possible values of the «frequencies» Ω where the boundary conditions of motion are given, is an «Eigenwert» problem. By solving it one will obtain a spectrum of proper frequencies Ω (which are complex values in the general case). This spectrum, generally speaking, contains separate, isolated, values («discrete spectrum») and also contains frequencies continuously filling the whole intervals of values («continuous spectrum»). It may be supposed that the frequencies of the continuous spectrum correspond to such motions $\vec{v}_1$ as are not damped at infinity, while the frequencies of the discrete spectrum correspond to motions which are damped at infinity rather rapidly (as is the case in many others Eigenwert problems).

For the problem of steadiness of the principal motion those of the frequencies $\Omega = \omega + i\gamma$ (ω, γ are real) are relevant in which the imaginary part is negative ($\gamma < 0$). The presence of such proper frequencies in the spectrum indicates the unsteadiness of the principal motion with respect to infinitely small perturbations. Such values of Ω are only possible among the frequencies of the discrete spectrum. In fact, the principal motion presents at infinity a plane-parallel homogeneous flow (we mean a flow past a body of finite dimensions). In so far as a plane-parallel flow is in no case steady, it will be evident that any perturbation that fails to disappear at infinity must necessarily be damped in time, or, in other words, correspond to frequencies Ω with $\gamma > 0$.

Accordingly, only the Ω frequencies of the discrete spectrum can be considered below.

In the case of sufficiently small velocities the principal motion is a steady one (inasmuch as a resting fluid is in any case steady). On the other hand, with sufficiently large Reynolds' numbers the laminar flow past a body is unsteady at any rate. In fact, with large Reynolds' numbers the motion far away from the body is not appreciably different from a plane-parallel flow unless in the region of the narrow «track». Now it follows from Lord Rayleigh's work that no motion with two-dimensional velocity distribution of such a type is steady, and one may expect that the same is true of the three-dimensional track.

If the value of the proper frequencies Ω are taken to be functions of Reynolds' number of the principal motion, then the critical value Re_{cr} is determined by the fact that for $\mathrm{Re}=\mathrm{Re}_{cr}$ the imaginary part of one of the frequencies Ω will vanish; suppose this frequency to be $\Omega_1=\omega_1+i\gamma_1$. For $\mathrm{Re}>\mathrm{Re}_{cr}$ we have $\gamma_1>0$; for such Reynolds' numbers as are near to the critical value Re_{cr}, γ_1 is small in comparison with ω_1. However, the expression $(1-2)$ for the respective function $\vec{v}_1(x, y, z, t)$ (with $\Omega=\Omega_1$) is only true for a very brief interval of time, as measured from the instant at which the stationary regime is broken. This is owing to the fact that the factor $e^{\gamma_1 t}$ grows rapidly with time. As a matter of fact, the modulus $|A|$ of the amplitude of non-stationary motion does not increase infinitely, but rather tends to a certain limit. With Re near to Re_{cr} (Re is always supposed to be greater than Re_{cr}), this limit is yet very small, too, and for determining it one may proceed as follows.

For very small times, when (2) is still applicable, we have

$$\frac{d|A|^2}{dt}=2\gamma_1|A|^2$$

In substance this expression is but the first term of a series of powers of A and A^*. With the increase of the modulus $|A|$ the subsequent three terms of this series must be taken into account. The next terms are terms of the third order. We, however, are interested not in the exact value of the differential quotient $d|A|^2/dt$, but in its mean value with respect to time, the averaging being made over time intervals that are large in comparison with the period $2\pi/\omega_1$ of the periodic spectrum $e^{-i\omega_1 t}$ (as $\omega_1 \gg \gamma_1$, this period is small compared to the time $1/\gamma_1$ during which the modulus $|A|$ changes appreciably). But the terms of the third order involve a periodic spectrum, and so they are eliminated upon averaging. (Strictly speaking, they do not vanish altogether, but yield quantities of order four; these quantities are supposed to be included into the terms of the fourth order). The terms of the fourth order include a term proportional to $A^2A^{*2}=|A|^4$; this term is not eliminated by averaging. Thus, up to terms of the fourth order we have

$$\overline{\frac{d|A|^2}{dt}}=2\gamma_1|A|^2-\alpha|A|^4 \qquad (3)$$

Here α is a positive constant (the case of negative α is considered below).

There are no signs of averaging over $|A|^2$ and $|A|^4$, because this operation is carried out over such time intervals as are small in comparison with $1/\gamma_1$. For the same reason in solving this equation we must disregard the dash over the derivative in the left-hand member. The solution of the equation (3) has the form

$$\frac{1}{|A|^2}=\frac{\alpha}{2\gamma_1}+\mathrm{const}\cdot e^{-2\gamma_1 t}$$

i. e. $|A|^2$ tends asymptotically to a limit

$$|A|^2_{\max} = 2\gamma_1/\alpha \tag{4}$$

γ_1 is a function of Reynolds' number; it vanishes with $\mathrm{Re} = \mathrm{Re}_{cr}$. Therefore for small $\mathrm{Re} - \mathrm{Re}_{cr}$ we have $\gamma_1 = \text{const} \cdot (\mathrm{Re} - \mathrm{Re}_{cr})$. Substituting this in (4) we shall see that

$$|A|_{\max} \sim \sqrt{\mathrm{Re} - \mathrm{Re}_{cr}} \tag{5}$$

Thus, the unsteadiness of the laminar motion for $\mathrm{Re} > \mathrm{Re}_{cr}$ leads to the appearance of a non-stationary periodic motion. When Re are close to Re_{cr}, this motion can be represented as a superposition of a stationary motion $\vec{v}_0(x, y, z)$ over a periodic motion $\vec{v}_1(x, y, z, t)$, having a small but finite amplitude which varies with Re directly as $\sqrt{\mathrm{Re} - \mathrm{Re}_{cr}}$. The velocity distribution in this motion has therefore the form

$$\vec{v}_1 = \vec{f}(x, y, z)\, e^{-i(\omega_1 t + \beta_1)} \tag{6}$$

where β_1 is a constant initial phase. When the differencies $\mathrm{Re} - \mathrm{Re}_{cr}$ are large, there is no longer any sense in separating the velocities into two parts $\vec{v}_0$ and $\vec{v}_1$. Here we have to deal simply with a periodic motion of frequency ω_1. If instead of the time the phase $\varphi_1 \equiv \omega_1 t + \beta_1$ is used as the independent variable, the function $\vec{v}(x, y, z, \varphi_1)$ may be said to be a periodic function of φ_1 with a period 2π, but no simple trigonometric function. It can be represented as a Fourier series

$$\vec{v} = \sum_p \vec{A}_p(x, y, z)\, e^{-i\varphi_1 p} \tag{7}$$

(the summation is carried out over all positive and negative integers p).

The essential fact is that only the absolute value of the factor, but not its phase are determined by the equation (3). The phase φ_1 remains in substance indefinite and depends upon the initial conditions which are a matter of chance and may cause β_1 to take any value. It will be obvious that the periodic motion under consideration is not determined uniquely by the given stationary boundary conditions of motion; one quantity, the phase, remains arbitrary. This motion may be said to have one degree of freedom, whereas stationary motion is completely determined by the given boundary conditions, and enjoys not a single degree of freedom.

As Re is further increased, this periodic motion, too, eventually becomes unsteady. The investigation of its unsteadiness should be conducted in a manner analogous to that described above. The rôle of the principal motion is now played by the periodic motion $\vec{v}_0(x, y, z, t)$ of frequency ω_1. Substituting $\vec{v} = \vec{v}_0 + \vec{v}_2$ with small $\vec{v}_2$ into the equation of motion, we shall again obtain for $\vec{v}_2$ a linear equation, but this time the coefficients of this equation are not only functions of the coordinates, but of time also; with respect to time, they are periodic functions with a period $\frac{2\pi}{\omega_1}$. The solution of such an equation should be sought in the form $\vec{v}_2 = \Pi(x, y, z, t)\, e^{-i\Omega t}$ where $\Pi(x, y, z, t)$ is a periodical function of time $\left(\text{with a period } \frac{2\pi}{\omega_1}\right)$. Unsteadiness sets in again when the frequency $\Omega_2 = \omega_2 + i\gamma_2$ turns up whose imaginary part γ_2 is positive and the corresponding real part ω_2 determines then the newly appearing frequency.

The result is a quasi-periodic motion characterized by two different periods It involves two arbitrary quantities (phases), *i. e.* has two degrees of freedom.

In course of the further increase of the Reynolds number new and new periods appear in succession, and the motion assumes an involved character typical of a developed turbulence. For every value of Re the motion has a definite number of degrees of freedom; in the limit, as Re tends to infinity, the number of degrees of freedom becomes likewise infinitely large. With n degrees of freedom the velocity distribution is described by expression of the type

$$\vec{v}(x, y, z, t) = \sum_{p_1, p_2, \ldots, p_n} \vec{A}_{p_1, \ldots, p_n}(x, y, z)\, e^{-i \sum_{i=1}^{n} p_i \varphi_i t} \qquad (8)$$

(summation over all integral numbers $p_1, p_2, \ldots, p_n$) where the phases are $\varphi_i = \omega_i t + \beta_i$; it contains n arbitrary initial phases β_i. The frequencies ω_i being incommensurable, it will be apparent that during a sufficiently long interval of time the fluid will pass through the states which are as close as we will it to a state set beforehand by choosing freely a set of simultaneous values for the phases φ_i. It should, of course, be borne in mind that the states whose phases differ only by a multiple of 2π are identical physically. So a turbulent motion is to a certain extent a quasi-periodical motion.

A somewhat different character the setting-up of a turbulent regime has in those exceptional cases (the Poiseuille motion and oth.) where the laminar motion remains stable with respect to infinitesimal perturbations, no matter how large are the Reynolds numbers. If the latter are sufficiently small, no unstationary motion is possible here at all; a steady unstationary motion becomes possible only after a certain value of Re is reached, which is here in the nature of a critical value. With very large Reynolds' numbers, the stationary motion may, notoriously, be materialized only if one is careful enough in eliminating the perturbations superimposed upon the motion. Contrary to this, if Re are close to Re_{cr}, the non-stationary motion is difficult to materialize. It may be thought therefore that the true value of Re_{cr}, say, in the case of Poiseuille's motion, lies in any case below the value generally adopted at present. As for the properties of the turbulent motion that appears here with $\mathrm{Re} > \mathrm{Re}_{cr}$, it should, contrary to the preceding case, enjoy from the outset a large number of degrees of freedom.

Finally, in principle, there is one more possible type of the loss of steadines by the laminar motion; this corresponds to the case where the coefficient before $|A|^4$ in (3) is positive, so that

$$\frac{d|A|^2}{dt} = 2\gamma |A|^2 + \alpha |A|^4$$

with positive α. If Re are somewhat smaller than Re_{cr}, the term of the second order is negative (since $\gamma_1 < 0$ for $\mathrm{Re} < \mathrm{Re}_{cr}$). But, the term of the fourth order being positive, the derivative $\frac{d}{dt}|A|^2$ will become positive when the amplitude of $|A|$ is sufficiently large. This means that the motion becomes steady with respect to sufficiently large perturbations even for $\mathrm{Re} < \mathrm{Re}_{cr}$. Thus, this type of unsteadiness is characterized by the fact that for a certain value of Re_{cr} of the Reynolds number the motion becomes unsteady with respect to infinitesimal disturbances, but even with $\mathrm{Re} < \mathrm{Re}_{cr}$ there is unsteadiness in response to perturbation of a finite magnitude. In this case along with a ove-mentioned critical Reynolds' number there should exist another, «lower» number which determines the instant of appearance of stable non-stationary solutions of the equations of motion.

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Academy of Sciences of the USSR.

Received
26. I. 1944.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 8

PHYSICS

ON THE MAXIMAL ENERGY ATTAINABLE IN A BETATRON

By D. IWANENKO and I. POMERANCHUK

(Communicated by V. A. Fock, Member of the Academy, 27.III.1944)

By means of a recently constructed induction accelerator, betatron, Kerst succeeded to obtain electrons up to 20 MeV(1). The principle of operation of the betatron is the acceleration of electrons by a tangential electric field produced by a changing magnetic flux which is connected with the magnetic field keeping electrons on the orbit by a simple relation. In contrast to cyclotron, whose applicability is essentially limited to the non-relativisitic region because of the defocussing of orbits due to the change of mass at high energies, there is no such limitation for the betatron.

We may point out, however, that another circumstance of a different nature would lead as well to the existence of a limitation for maximal energy attainable in a betatron. This is the radiation of electrons in the magnetic field. Indeed, electrons moving in a magnetic field are accelerated and must radiate in accordance with the classical electrodynamics.

One can easily see that quantum effects do not play here any important rôle, as the dimension of the orbit is very great.

As was shown by one of us (2), an electron moving in a magnetic field H radiates per unit of path the energy

$$-\frac{dE}{dx}=\frac{2}{3}\left(\frac{e^2}{mc^2}\right)^2\left(\frac{E}{mc^2}\right)^2\left[\frac{\vec{v}\vec{H}}{c}\right]^2 \qquad (1)$$

(e, charge; m, mass; $\vec{v}$, velocity; E, energy of the electron. E is assumed much greater than mc^2).

In the betatron v is normal to H and practically for the whole path equal to c.

Then we have

$$-\frac{dE}{dx}=\frac{2}{3}r_0^2\left(\frac{EH}{mc^2}\right)^2 \qquad \left(r_0=\frac{e^2}{mc^2}\right) \qquad (2)$$

The limiting value of energy E_0 is to be determined from the condition that the radiated energy (2) will be equal to energy gained by the electron in the electric field produced by magnetic flux per unit of path (3):

$$\frac{2}{3}r_0^2\left(\frac{E_0H}{mc^2}\right)^2=\frac{e\left|\frac{d\Phi}{dt}\right|}{2\pi R_0 c}=eR_0|\dot{H}|, \quad \dot{H}=\frac{dH}{dt} \qquad (3)$$

(R_0, radius of the orbit; Φ, induction flux).

Hence

$$E_0 = mc^3 \sqrt{\frac{3}{2}\frac{e}{c}\left|\frac{R_0\dot{H}}{r_0^3 H^2}\right|} \qquad (4)$$

Taking lor H, $\dot{H}$, and E the values used up to the present, we get $E_0 \approx 5 \cdot 10^8$ eV, which is only five times as great as the energy which one expects to obtain in the betatron which is now in construction.

From (4) one sees that E_0 varies inversely as the magnetic field applied and directly as the square root of energy gained in the rotational electric field per unit path.

All this requires the using of smaller H or of higher frequencies with the purpose of getting higher limiting values of E_0.

The radiative dissipation of energy of electrons moving in a magnetic field must be also of importance for the discussfon of the focussing of the electronic beam, as the energy of accelerated particles will grow more slowly with the growth of H if the radiation is taken into account. This latter question may deserve a separate discussion.

It may be also of interest to investigate the use of a betatron as a generator of the electromagnetic radiation of the indicated type.

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Received
20.I.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 8

GEOPHYSICS

COLOUR TEMPERATURE INDEX OF THE SKY ZENITH AT LOW POSITIONS OF THE SUN OVER THE HORIZON

By N. N. KALITIN

(Communicated by S. I. Wawilow, Member of the Academy, 20.I.1944)

A number of works on the colour temperature of the sky carried out recently (1) show that the method of colour temperature can well be applied to the study of the state of atmosphere as a turbid medium. Changes of the optical state of the atmosphere are brought out with especial clearness when measurements of colour temperature are made not for the sky as a whole, but only for its zenith region.

In order to determine the dependence of colour temperature at the zenith of a cloudless and an overcast sky upon the height of the Sun above the horizon some observations were carried out near Sverdlovsk (Vysokaya Dubrava, 300 m above sea level).

A tube 65 mm across and 150 mm in length blackened on the inside had a sulphur-selenic photocell fixed at its sealed end. The photocell designed by K. G. Trofimov was covered with a red filter, the optic angle of the photocell from the centre of the latter being 20°. At the end of another similar tube was fixed a photocell of the same kind, but covered with a blue (combined) filter. Both these tubes could be directed with their open ends to any point of the sky.

From the spectral sensitivity of the photocell and the spectral characteristics of the filters it has been determined that the photocell with the red filter was sensitive to a radiation from 6000 to 6800 Å, that with the blue one, from 3000 to 4500 Å. Thus, the second photocell is sensitive not only to the red but to the violet and ultra-violet radiations as well; in the following this filter will for brevity be said to be sensitive to blue radiation.

With the aid of an additional liquid filter—a $CuSO_4$ aqueous solution—it was shown that the blue filter used in the experiment let through neither the red nor the infra-red radiation.

The two tubes with photocells and filters were mounted close side-by-side; during the observations they were directed to the zenith. The photocurrents were measured by means of a Zeiss loop galvanometer. During the observations the set was protected from the vertical Sun rays by a screen, owing to which the photocells reacted only to the scattered radiation of the zenith region of the sky within an angle of 20°.

The colour temperature was characterized by the ratio of the galvanometer readings taken with the blue filter to that taken with the red one.

The sensitivity of the set made it possible to start observation when the Sun was 1—2° below the horizon.

During the spring—autumn period of 1943 several dozens of sets of observations were obtained both with a perfectly cloudless sky and with a sky covered with clouds of low forms: Stratus (St), Stratus-nimbus (NS), Stratus-cumulus (Sc).

From an analysis of the data obtained it may be seen that as the Sun approaches the horizon the colour temperature in the case of cloudless sky decreases and the scattered radiation of the atmosphere grows redder, owing to a stronger decrease of the blue radiation. If the sky is completely covered with clouds of the forms mentioned above, an increase of the colour temperature is observed because of the «blue» radiation increasing at the expense of the red one.

Table 1 illustrates two sets of observations made with clear and overcast sky.

Table 1

14.III. 1943, cloudless sky				11.IX.1943, cloudiness 10			
Sun altitude	Colour temperature	Sun altitude	Colour temperature	Sun altitude	Colour temperature	Sun altitude	Colour temperature
30°.0	2.14	6°.3	1.85	35°.3	0.69	6°.9	0.75
29.5	2.09	5 .6	1.87	35 .0	0.66	5 .6	0.77
27.7	2.17	5 .0	1.79	34 .0	0.65	4 .1	0.83
25.8	2.16	4 .0	1.75	33 .3	0.69	2 .9	0.79
23.1	2.22	3 .4	1.68	29 .0	0.72	2 .2	0.89
20.4	2.19	2 .8	1.61	26 .2	0.70	1 .5	1.03
17.2	2.16	2 .2	1.58	25 .1	0.66	0 .9	1.17
13.7	2.14	1 .7	1.54	23 .7	0.71	0 .6	1.09
12.3	2.16	1 .2	1.55	17 .8	0.72	0 .3	1.18
10.6	2.17	0 .8	1.50	15 .0	0.72	0 .0	1.27
7.4	1.99	0 .5	1.52	12 .4	0.73	—0 .4	1.79
6.8	1.94	0 .0	1.44	10 .3	0.74	—0 .8	2.10
				8 .4	0.72		

On the 14th of March, with a cloudless sky colour temperature was first found to increase along with the lowering of the Sun, reaching its maximum with the position of the Sun about 20° above the horizon, and sharply to diminish as the Sun approached the horizon, dropping as low as 1.44 from the maximum of 2.22.

All the series obtained with cloudless sky and good atmosphere transparency show an analogous course, *i. e.* the minimum values of colour temperature are observed with the Sun close to the horizon, strongly increasing with the rise of the Sun, and reaching they maximum when the Sun is 15—20° high, after which they begin gradually to diminish.

Such a course can readily be explained: with the decline of the Sun, the Sun rays have to travel through a larger mass of atmosphere before they reach the zenith; the short-wave radiation becomes much less intense than the red one; if the turbidity of the atmosphere remained constant throughout the observation period, we should observe a «reddening» of the scattered radiation of the atmosphere, followed by a decrease in colour temperature.

At small Sun altitudes the short-wave component of illumination decreases at a higher rate than the long-wave one, which is evident from the upper part of Table 2 showing the galvanometer readings for the observations made on March 15th with a cloudless sky, with Sun altitudes from 8°.2 to—0°.3. From these readings it may be seen that when the Sun was at 8°.2, the blue radiation was by 84 per cent stronger than the red one, while it was only by 40 in excess with 0°.0, decreasing more than twice as compared with the red one.

The decrease of the colour temperature beginning with the Sun height of 15—20° and higher may be explained by the fact that, first, with the increase of the height of the Sun the interval between the Sun and the zenith decreases (in degrees), which is necessarily followed by a reduction in the colour temperature; secondly, owing to the ascending currents the atmosphere becomes towards midday contaminated with dust and condensation products (condensation turbidity), which calls forth a decrease in the colour temperature value.

The author observed several cases where «directly in his sight» the appearance in a cloudless sky of condensation turbidity at the zenith was followed by a drop

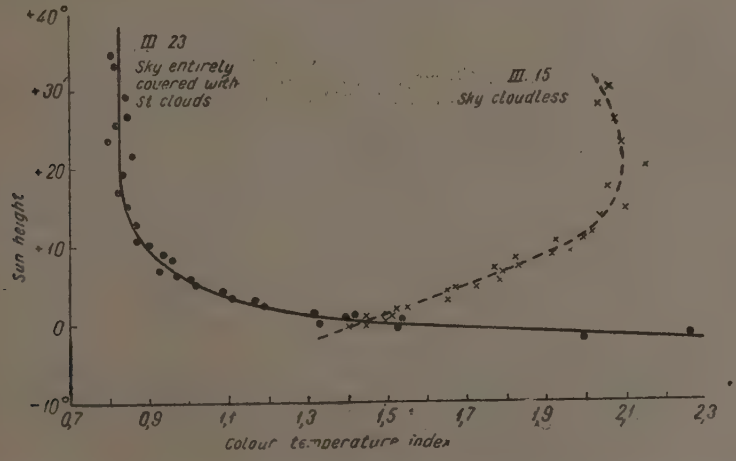
Table 2

15.III.1943, cloudless sky									
Sun altitude . . .	8°.2	6°.4	4°.4	3°.0	2°.2	1°.0	0°.3	0°.0	—0°.30
Red	26 .4	21 .9	16 .2	12 .9	11 .4	7 .5	4 .50	3 .30	3 .0
Blue	48 .6	39 .0	27 .0	21 .3	17 .7	11 .4	6 .90	4 .80	4 .20
30.III.1943, cloudiness 10									
Sun altitude . . .	8°.5	6°.0	4°.0	2°.5	1°.2	0°.6	0°.0	—0°.6	—1°.3
Red	60 .6	55 .5	27 .0	11 .7	6 .2	3 .00	1 .89	0 .96	0 .72
Blue	51 .9	46 .5	27 .3	16 .6	9 .2	4 .90	3 .00	2 .16	1 .56

in the colour temperature. The second part of Table 1 shows that in the case of a solid cloud-cover the dependence of the colour temperature value on the position of the Sun is quite different. Within the range of large Sun altitudes up to 15—20° the colour temperature remains constant irrespective of the position of the Sun; but as the Sun approaches the horizon, a strong increase in the colour temperature is observed, which is especially pronounced at small heights.

In this case the increase in the colour temperature with the decline of the Sun may be put down to the long waves decreasing at a higher rate as compared with the short ones. The bottom part of Table 2 shows that for heights of the Sun equal to 8°.5, red radiation exceeds the blue one by 17 per cent, and when the Sun is at the horizon, it decreases by 54 per cent.

The run of the colour temperature in the two extreme cases, *i. e.* with a cloudless sky and a sky completely covered with low forms of clouds, is obvious from the figure. No explanation can for the present be given of the opposite run of the co-



lour temperature when the sky is covered with clouds. The process is essentially the same but for the presence of layer of clouds at a moderate height above the Earth's surface.

It is a matter for regret that the data available on radiation of different wavelengths being let through and scattered by clouds (mists) are very sparse and contradictory (2); they cannot therefore be had recourse to in explaining the phenomenon observed. But the very fact of an inverse run of the curve of colour temperature in the case of clouds is beyond any doubt, for it occurs in all the observations.

It should be pointed out that even with a cloudless sky the colour temperature increases with the decline of the Sun instead of decreasing if the atmosphere is very turbid. Such a run of the colour temperature expressed moreover with particular clearness I happened to observe in the evening of August 30. From the records made it may be seen that at a position of the Sun at 8° and higher over the horizon, the sky assumed a pale-yellow hue. There were cases where the colour temperature of a cloudless sky showed almost no change with the position of the Sun. So it would seem that, according to the degree of turbidity (transparency) of the atmosphere in the case of a cloudless sky, the colour temperature may exhibit every possible kind of variation with the Sun's altitude. It may be found to decrease with the height of the Sun as well as to increase after the Sun has passed the neutral position. The run of the colour temperature in such cases may possibly serve as an index of the turbidity of the atmosphere. In order to explain and check up the run of the curve of colour temperature, further work, both theoretical and experimental, is needed.

Observations of colour temperature carried out with a sky completely covered with clouds of low stages showed that at Sun altitudes exceeding 30°, when the value of colour temperature does not depend any longer on the height of the Sun, it changes from 0.50 to 0.90 for the clouds of the St group, being now strictly dependent on the thickness of the cloud layer. This makes us believe that the value of the colour temperature may serve to evaluate the thickness of the cloud layer (or the mass of the fog) as well as the changes in their thickness. I have at my disposal a number of serial observations in which successive measurements of colour temperature with St clouds show it to have been increasing gradually; in all cases the cloud-cover was thinning out and ultimately disappeared altogether.

This result is of great practical importance; if *e. g.* successive observations made in the fog show an increase in colour temperature, this may serve as an indication that the fog is thinning out and is likely to vanish soon, and *vice versa*.

Received
20. I. 1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 8

OCEANOLOGY

ON THE DETERMINATION OF THE MIXING COEFFICIENT IN THE SEA BY MEANS OF ϑ, s-CURVES

(ON THE CORRECTNESS OF JACOBSEN'S METHOD)

By W. B. STOCKMANN

(Communicated by P. P. Shirshov, Member of the Academy, 6.XII.1943)

The theorems on the geometrical properties of the ϑ, s-curves as formulated by the author earlier (1) make it possible also to examine critically the theoretical foundation of Jacobsen's method (2) for determining the mixing coefficient in the sea by means of ϑ, s-curves. This generally recognized method is known to consist essentially in choosing one of the two ϑ, s-curves, *viz.* that corresponding to the later instant, and to draw at its point of maximum curvature a tangent which crosses the other ϑ, s-curve (corresponding to the earlier instant) at two points with known values of the parameter z' (depth), *viz.* z' and z''. The mixing coefficient k is computed according to Jacobsen from the formula

$$k = \frac{(z' - z'')^2}{8\Delta t} \qquad (1)$$

in which Δt denotes the time interval separating the instances of observation.

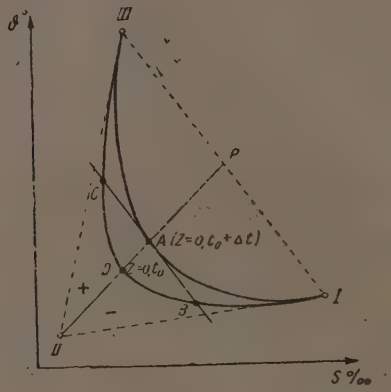
However, not to speak of the fact that it is rather difficult in practice to draw a tangent at the point of maximum curvature in a ϑ, s-curve, it must be borne in mind that the curvature is markedly variable with the scale of temperature and salinity adopted in plotting the diagram. It is apparent that the position of the maximum curvature point upon such a deformed drawing fails to correspond to the position of this point in natural coordinates. This important circumstance is entirely lost sight of in both the theory of the method and in its application to practice.

What is more, a closer study of the theory of Jacobsen's method reveals that it remains an unsettled question, whether it is really essential to draw the tangent just at the point of maximum curvature and not at some other point of the curve. In the light of these preliminary observations a more rigorous proof of Jacobsen's method appears to be quite indispensable. It is well to note that the starting statements in the theory of this method correspond fully to the conditions of the problem considered in our previous note.

The author's purpose now is to show that Jacobsen's method, in so far as its geometrical side is concerned, involves, in substance, a very serious error. However, the deduction of Jacobsen's formula (1) as made by the present author on the basis of the geometrical properties of ϑ, s-curves he has established, makes it possible to obviate this kind of error and to determine the value of the mixing coefficient with sufficient accuracy.

For this purpose let us consider two ϑ, s-curves (see figure).

One of them is supposed to correspond to an instant t_0, and the other, to an instant $t_0+\Delta t$. At the point of intersection A of the main median line $II\ P$ of the mixing triangle $I\ II\ III$ with the second ϑ, s-curve we draw a straight line parallel to side $I\ III$ of the mixing triangle. By the established theorem the straight line is tangent at point A to the second ϑ, s-curve and



will cross the first curve at points B and C, at which the parameter z takes values which are equal but opposite in sign to the values it takes at points $A(z=0,\ t_0+\Delta t)$ and $D(z=0,\ t_0)$ lying upon the main median of the mixing triangle.

Suppose, in accordance with what has been said earlier, that the parameters of the points B and C are characterized by the values $B\left(-\frac{\Delta z}{2},\ t_0\right)$; $C\left(\frac{\Delta z}{2},\ t_0\right)$.

Consequently, Δz represents the difference between the values of the parameter at points B and C.

Suppose, further, that temperature and salinity at points A and B can be expressed with an accuracy sufficient for practical purposes in the form of a sum of the first terms of a Taylor series

$$\left.\begin{aligned} \vartheta_A &= \vartheta_0 + \frac{\partial \vartheta}{\partial t}\Delta t; \quad s_A = s_0 + \frac{\partial s}{\partial t}\Delta t \\ \vartheta_B &= \vartheta_0 - \frac{\Delta z}{2}\left(\frac{\partial \vartheta}{\partial z}\right)_0 + \frac{(\Delta z)^2}{8}\left(\frac{\partial^2 \vartheta}{\partial z^2}\right)_0; \quad s_B = s_0 - \frac{\Delta z}{2}\left(\frac{\partial s}{\partial z}\right)_0 + \frac{(\Delta z)^2}{8}\left(\frac{\partial^2 s}{\partial z^2}\right)_0 \end{aligned}\right\} \quad (2)$$

where the subscript 0 indicates values referring to the central region of the intermediate mass of water ($z=0$).

The equation of the tangent BC can be written in the form

$$\vartheta_A - \vartheta_B = \left(\frac{d\vartheta}{ds}\right)_0 (s_A - s_B) \quad (3)$$

Substituting in (3) the values from (2) and transposing the terms, we obtain

$$\left[\frac{\partial \vartheta}{\partial t} - \left(\frac{d\vartheta}{ds}\right)_0 \left(\frac{\partial s}{\partial t}\right)\right]\Delta t = -\frac{\Delta z}{2}\left[\left(\frac{\partial \vartheta}{\partial z}\right)_0 - \left(\frac{d\vartheta}{ds}\right)_0 \left(\frac{\partial s}{\partial z}\right)_0\right] + \frac{(\Delta z)^2}{8}\left[\left(\frac{\partial^2 \vartheta}{\partial z^2}\right)_0 - \left(\frac{d\vartheta}{ds}\right)_0 \left(\frac{\partial^2 s}{\partial z^2}\right)_0\right] \quad (4)$$

As we have for a ϑ, s-curve the relation

$$\left(\frac{d\vartheta}{ds}\right)_0\left(\frac{\partial s}{\partial z}\right)_0=\left(\frac{\partial\vartheta}{\partial z}\right)_0 \qquad (5)$$

the first expression in brackets on the right-hand side of (4) will vanish. Differentiating (5) with respect to z as a complex function, we obtain

$$\left(\frac{\partial^2\vartheta}{\partial z^2}\right)_0=\left(\frac{d^2\vartheta}{ds^2}\right)_0\left(\frac{\partial s}{\partial z}\right)_0^2+\left(\frac{d\vartheta}{ds}\right)_0\left(\frac{\partial^2 s}{\partial z^2}\right)_0 \qquad (6)$$

Hence

$$\frac{(\Delta z)^2}{8}\left[\left(\frac{\partial^2\vartheta}{\partial z^2}\right)_0-\left(\frac{d\vartheta}{ds}\right)_0\left(\frac{\partial^2 s}{\partial z^2}\right)_0\right]=\frac{(\Delta z)^2}{8}\left(\frac{d^2\vartheta}{ds^2}\right)_0\left(\frac{\partial s}{\partial z}\right)_0^2. \qquad (7)$$

If we take into account the differential equations of Fourier and Fick

$$\frac{\partial\vartheta}{\partial t}=k\frac{\partial^2\vartheta}{\partial z^2};\ \frac{\partial s}{\partial t}=k\frac{\partial^2 s}{\partial z^2} \qquad (8)$$

which describe the process of vertical conduction and diffusion of heat, it is no difficult matter to show, by combining (8) with (5) and (6), that

$$\frac{\partial\vartheta}{\partial t}-\frac{d\vartheta}{ds}\frac{\partial s}{\partial t}=k\frac{d^2\vartheta}{ds^2}\left(\frac{\partial s}{\partial z}\right)^2$$

Substituting the last expression in place of that included into brackets on the left-hand side of (4), and omitting the subscript 0, we get

$$k\frac{d^2\vartheta}{ds^2}\left(\frac{\partial s}{\partial z}\right)^2\Delta t=\frac{(\Delta z)^2}{8}\left(\frac{d^2\vartheta}{ds^2}\right)\left(\frac{\partial s}{\partial z}\right)^2$$

or

$$k=\frac{(\Delta z)^2}{8\Delta t} \qquad (9)$$

This formula is seen to be identical with Jacobsen's formula (1).

The essential feature of the author's way to deduce Jacobsen's formula is that the tangent BC is not drawn through the point of maximum curvature of the second ϑ, s-curve (whose position is, generally speaking, indefinite), but at the point of intersection of the main median of the mixing triangle with the second ϑ, s-curve.

The latter point does not, generally speaking, coincide with the point of maximum curvature of this curve. Such a coincidence takes place only in a particular case where the mixing triangle is isosceles, with equal sides $I\,II$ and $II\,III$.

In so far as in this case the point of maximum curvature is on the main median (bisectrix) of the mixing triangle, the two constructions are identical. It is just this particular case that has been considered by Jacobsen who believes himself, however, to have obtained a universal method of determining the coefficient k.

He is in error because his theory ignores so important an element as the mixing triangle. Contrary to the author's principle, Jacobsen has not thought it necessary to limit the branches of the ϑ, s-curve, which, as a matter of fact, must not reach beyond the points corresponding to the position of the extreme water masses. Naturally, this important circumstance cannot but essentially affect the position of the intermediate points of the curve, each of which corresponds a definite value of the parameter z. Having excluded from his field of vision the totality of water masses, Jacobsen was left the axis of symmetry of the ϑ, s-curve as his only «bearings», because he had failed to limit its branches.

In order completely to clear the question as to the rigorousness of Jacobsen's method, one has yet to explore the possibility of transforming any mixing triangle formed by arbitrary water masses into an isosceles triangle.

Should such a transformation be always possible by merely changing the scale along the orthogonal axes ϑ, s, Jacobsen's construction might then, of course, be always brought in correspondence with the author's construction and both methods would give the same results, irrespective of the scale chosen.

If the coordinates of the apexes *I*, *II*, *III* in a given mixing triangle be denoted by (s_1, ϑ_1); (s_2, ϑ_2); (s_3, ϑ_3), respectively, one will have for answering the proposed question to investigate the following indefinite equation in which the two members represent the length of the sides $I-II$ and $II-III$ of the triangle:

$$\alpha^2 (x_1 - x_2)^2 + \beta^2 (y_1 - y_2)^2 = \alpha^2 (x_3 - x_2)^2 + \beta^2 (y_3 - y_2)^2$$

where α and β designate the proportionality coefficients of the scales along the x and y (s and ϑ) axes, respectively.

The last equation is reducible to the form

$$\alpha^2 A + \beta^2 B = 0$$

where

$$A = (x_1 - x_2)^2 - (x_3 - x_2)^2; \; B = (y_1 - y_2)^2 - (y_3 - y_2)^2$$

Consequently, α should be linked with β by the relation

$$\alpha = \beta \sqrt{-\frac{B}{A}}$$

It appears from the results obtained that the transformation into an isosceles triangle is possible and can be materialized in many ways, only if A and B have opposite signs. Otherwise the transformation is impossible. This is why Jacobsen's construction is not correct in general; contrary to this, the author's construction, by virtue of its invariance with respect to scale transformation in ϑ, s-curve, is the only one that will meet all cases, no matter whether the given mixing triangle may not be transformed into an isosceles triangle.

Thus, two methods for computing the mixing coefficient k are possible. In computing k according to formula (9) the tangent must be drawn at the point of intersection of the ϑ, s-curve with the main median of the mixing triangle, and if the mixing coefficient is computed by means of formula (6) [1], two tangents are to be drawn at the points of intersection of the lateral median of the triangle with two consecutive ϑ, s-curves. It must also be emphasized that (9) is an approximate formula, while (6)[1] is a rigorous formula.

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Received
26 X. 1943.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 8

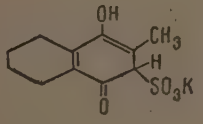
CHEMISTRY

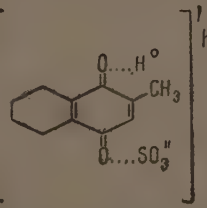
ON THE STRUCTURE OF THE BISULPHITE COMPOUND OF 2-METHYL-1,4-NAPHTHOQUINONE AND OF ITS 3-SULPHONIC ACID (ANALOGUES OF VITAMIN K_1)

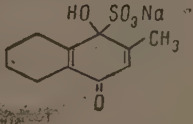
By V. N. UFIMTZEV

(Communicated by A. E. Poray-Koshitz, Member of the Academy, 29.II.1944)

Numerous attempts have been made lately to obtain water-soluble analogues of vitamin K_1. In particular, the reaction between 2-methyl-1, 4-naphthoquinone and solutions of the bisulphites of alkaline metals has been utilized. Various authors have suggested different structures for the product formed, and sometimes these suggestions were in disaccord with the usual conceptions of the structure of bisulphite compounds (4,5).

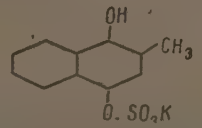
(I) 

(II) 

(III) 

The Moore (1) formula (I) which is a tautomeric form of 2-methyl-1, 4-naphthohydroquinone-3-sulphonic acid does not agree with the properties of the substance which differs from the latter compound by its easy cleavage into components in an alkaline medium and its stability towards the action of oxidants in acid medium.

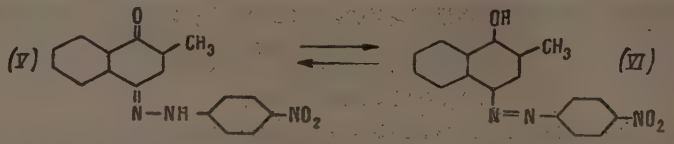
The formula of the «bisulphite complex» of Bochvar and Shemiakin (2) is, as a matter of fact, analogous with the structure of the potassium salt of 2-methyl-1, 4-naphthohydroquinone sulphate (IV).

(IV) 

Yet the behaviour of the product does not correspond at all to the properties of these esters known by their stability in alkaline medium and the easy hydrolysis of the ester sulpho-group in acid medium (3). As for the existence in an aqueous solution of this «bisulphite complex» of an equilibrium with five other compounds, among which three substances (the oxonium salt, the adduct 1 and the salt of naphthohydroquinone-3-sulphonic acid) have not been isolated as yet, it may be remarked that such signs as a yellowish or yellow colour of the solutions and the formation of traces of a precipitate under the action of reagents, weaken gradually upon purification, and even two crystallizations are sufficient for their complete disappearance.

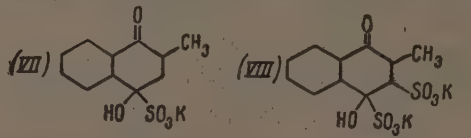
The pure product of the action of potassium bisulphite upon 2-methyl-1, 4-naphthoquinone, $C_{11}H_9O_5SK.H_2O$ (found: K 12.59; 12.61 per cent; H_2O 5.15; 5.51 per cent), when dissolved in water in the cold yields a thoroughly colourless solution and gives no precipitate with solutions of barium chloride and silver nitrate nor any traces of potassium 2-methyl-1, 4-naphthoquinone-3-sulphonate, after chlorine has been passed through the solution. Only when its aqueous solution had been left standing for long with barium chloride, did the formation of traces of barium sulphite take place, owing to the gradual hydrolysis of the bisulphite compound by the hydroxyl ions of water. This hydrolysis, characteristic of the usual bisulphite compounds of naphthols and of their azodyes, has been studied by Vorozhzov sen. (4).

The reaction product of potassium bisulphite with potassium-2-methyl-1, 4-naphthoquinone-3-sulphonate, $C_{11}H_8O_8S_2K_2$ (found: K 19.14; 19.03 per cent), shows similar properties, but when its aqueous solution remains standing for a short time



with barium chloride, there separates a copious white crystalline precipitate (prisms). This precipitate which, according to Bochvar and Shemiakin, might be expected to be barium sulphate, is in fact a poorly soluble barium salt of the bisulphite compound, $C_{11}H_8O_8S_2Ba.H_2O$ (found: Ba 27.74 per cent; H_2O 3.50 per cent).

Having oxidized the «bisulphite complex» of 2-methyl-1,4-naphthoquinone-3-sulphonic acid with hydrogen peroxide, Bochvar and Shemiakin have isolated and analysed the «bisulphate complex» of this compound. However, the data from which



they inferred the structure to this product are based only on the determination of the potassium content, the value for which proved to lie just between the values calculated for the «bisulphite» and the «bisulphate complexes». They have found: K 18.60 per cent, calculated for $C_{11}H_7O_5SK.KHSO_3$: K 19.06 per cent; for $C_{11}H_7O_5SK.KHSO_4$: K 18.34 per cent.

Having repeated their experiments we have found that part of the substance undergoes complete oxidative cleavage and the residue is the unchanged initial product (the bisulphite compound of 2-methyl-1, 4-naphthoquinone-3-sulphonate) which can be easily identified in the form of its barium salt described above.

The formula of Palladin (5) (III) represents the reaction as the addition of potassium bisulphite to the keto-group of the quinone. It agrees with the behaviour of the substance which, according to the opinion of Raschig (6), splits off easily its sulpho-group under the action of alkaline reagents. However, insignificant corrections should be introduced into this formula, since in 2-methyl-1, 4-naphthoquinone there are two keto-groups which can enter the reaction, and the reaction capacity of one of them (in position 1) should be considerably lower owing to the presence of the methyl-group in the *o*-position, according to the Kehrmann rule (7).

In order to check these conjectures *p*-nitro-phenylhydrazine was condensed with 2-methyl-1, 4-naphthoquinone. The compound (V) was obtained: 2-methyl-1, 4-naphthoquinone-4-(4'-nitro-phenyl)-hydrazone, $C_{17}H_{13}O_3N_3$ (m. p. 267—267.5°; found N 13.85 per cent) which proved to have identical properties with 4-(4'-nitro-benzene-azo)-2-methyl-1-naphthol (VI) (m.p. 267—268°) prepared by Lesser (8) by the copulation of *p*-nitro-phenyldiazonium chloride with 2-methyl-1-naphthol.

Thus, in 2-methyl-1,4-naphthoquinone the keto-group in position 4 is more active, and to the products of the reaction of potassium bisulphite with 2-methyl-, 1,4-naphthoquinone and with its 3-sulphonic acid the structure of the potassium salts of 1-oxo-2-methyl-4-hydroxy-1, 4-dihydronaphthalene-4-sulphonic acid (VII) and, correspondingly, 1-oxo-2-methyl-4-hydroxy-1,4-dihydronaphthalene-3,4-disulphonic acid (VIII) should be ascribed. Naturally, the bisulphite compound of 2-methyl-1, 4-naphthoquinone (VII) in which the keto-group in position 4 has already reacted with potassium bisulphite, fails to react with *p*-nitro-phenyl-hydrazine.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 8

PHYSICAL CHEMISTRY

THE IGNITION LIMITS OF A FULMINATING MIXTURE OF HYDROGEN AND OXYGEN IN AN IRON VESSEL

By A. B. NALBANDJAN

(Communicated by N. N. Semenov, Member of the Academy, 22.II.1944)

N. N. Semenov's theory of branching chains [1] is known satisfactorily to explain the presence of the ignition limits observed in a number of fulminating mixtures.

According to the theory, the upper limit of ignition should practically have one and the same value in the same temperature interval, regardless what the reaction vessel is made of, whereas the value of the lower limit is markedly dependent upon the material of which the vessel is made. Experiments carried out by Hinshelwood and his coworkers [2], Semenov and his school [1] and other investigators in pyrex, quartz and glass vessels fully corroborate the conclusion following from the theory of branching chains. In vessels made of other materials no experiments had been performed regrettably until 1938, except for an unsuccessful attempt to obtain the ignition limits in a silver vessel by Moelwyn-Hughes, Rolfe and Hinshelwood [3] who did not detect even the traces of a homogenous reaction in the temperature range in which it usually completely predominates.

As we have shown in an ignition study of a fulminating mixture in a silver vessel [4], this misunderstanding may now be regarded as elucidated. In 1938 Stern, Sokolik and Kravetz [5] carried out in a vessel made of stainless steel a series of experiments on the determination of the upper ignition limit of hydrogen-air mixtures, and also measured the dependence of the induction period within the region of self-ignition upon the temperature and pressure of the reacting mixture. In their experiments the investigators employing the method used by Mallard and Le Chatelier—direct introduction of the reacting mixture into a heated, evacuated vessel—obtained in the usual temperature range an unusually high upper limit, several times higher than the ignition limit measured in quartz or pyrex vessels. The most interesting point in the measurement of the lower limit of ignition, under conditions when the probability of active centres perishing on the walls of the reaction vessel is much greater than in glass vessels, was regrettably left untouched upon by the investigators.

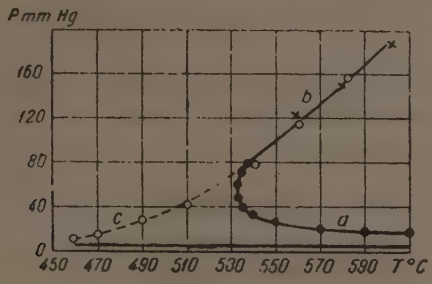
Stern, Sokolik and Kravetz results relating, in particular, to the upper limit of ignition, are regrettably unreliable owing to errors in method. From the standpoint of the theory of branching chains, highly important in a correctly set up experiment is a comparison of the limits of ignition obtained in glass vessels with those measured in an iron vessel.

It was with the aim of providing these conditions that we undertook the present investigation.

Procedure and Experimental Results. The reaction vessel was a stainless steel cylinder 110 mm long and 26 mm in diameter with a narrow

let-out tube. The change in pressure inside the vessel was registered by a membrane manometer.

By decreasing the volume of the supply vessel and increasing the opening in the stop-cock between the latter and the reaction vessel, we were able to reduce the time required to introduce the reacting mixture and equalize its pressure to 0.02 sec. Notwithstanding such a rapid introduction of the gas into the hot



evacuated reaction vessel, we succeeded in obtaining only a small section of the upper limit, represented by the points on the curve (see figure). All attempts to obtain the continuation of the upper limit did not meet with success. The mixture invariably ignited before reaching the upper limit.

A photographic study of the change in pressure from the moment the mixture was introduced into the evacuated vessel to the moment of the ignition, registered by the membrane manometer, showed that beginning with a certain pressure (at a constant temperature) the induction period of the reaction becomes much shorter than the time required to pass the mixture over into the reaction vessel. As a result, the mixture ignites before its pressure reaches the given value. These experiments graphically showed the limit of applicability of Mallard and Le Chatelier's method for determining the upper limit of ignition. To obtain the continuation of the upper limit, we resorted to the method of lowering the reaction vessel with the gaseous mixture at various pressures into a hot furnace*.

In our experiments we employed the following procedure: preliminarily heated up to 500° C the iron vessel with the mixture of $2H_2+O_2$ at a pressure higher than that corresponding to the upper limit, was rapidly lowered into a special oven with molten tin. Then, by turning the stop-cock connecting the reaction vessel with the pumps, the mixture was slowly evacuated until ignition occurred, accompanied by the characteristic crackle and spreading of the flames along a glass observation tube attached to the iron vessel. The maximum pressure at which the mixture still ignited was taken as the limiting pressure. In exactly the same way measurements were made of the upper limit at various temperatures. As the experiments showed, reproducibility is good.

By this method we measured the values of the upper limit of ignition up to 600° C. The values of the upper limit that were obtained experimentally are indicated on the figure by crosses. As is seen, the upper limit of ignition measured by lowering the reaction vessel into a hot furnace is a direct continuation of that part which we were able to determine by directly admitting the gas into the reaction vessel. For a comparison of the absolute value of the upper limit obtained in an iron vessel with that measured in a quartz vessel, we have reproduced a curve of the upper limit (indicated by circles) given by Hinshelwood and Williamson (2). As we see, starting with 540°C and higher both curves practically coincide, which fully substantiates the conclusion following from the theory of branching chains reg-

*It should be noted that this method is applicable only in those cases where the surface reaction is slight and, consequently, the reaction products (in this case, water) may only slightly affect the magnitude of the upper limit.

arding the practical independence of the upper limit of ignition upon the material and state of the reaction vessel.

The lower limit of ignition was measured by the method of direct gas introduction into the reaction vessel, and the values that we obtained are plotted in the figure (crosses) as a function of the temperature.

Below by a continuous line we have reproduced for comparison a curve of the lower limit measured in a quartz vessel by Hinshelwood and Williamson. As we see, the lower limit of ignition in an iron vessel is considerably higher than that measured in a quartz vessel, which can only be explained by the fact that the active centres of the reaction (H and O atoms, OH radicals, and others) recombine much easier on metallic surfaces.

The temperature dependence of the lower limit is well described by the law: $P_{crit.} = ae^{E/RT}$, where E is equal to 9—10 kcal.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
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PHYSICAL CHEMISTRY

ON THE THERMODYNAMICS OF INTERMETALLIC COMPOUNDS

(ON THE QUESTION OF BERTHOLLETIDES AND DALTONIDES)

By N. N. SIROTA

(Communicated by N. P. Chizhevsky, Member of the Academy, 24.II.1944)

The presence of intermediate intermetallic phases in binary and multicomponent equilibrium systems and the boundaries of their distribution are determined by the following condition of thermodynamic equilibrium (1) at a temperature and pressure constant and equal for all the phases in equilibrium:

$$\frac{\partial F_1}{\partial m_1}=\frac{\partial F_2}{\partial m_2}=\dots, \quad \frac{\partial F_1}{\partial n_1}=\frac{\partial F_2}{\partial n_2}=\dots \tag{1}$$

where $F_1, F_2, \dots$ are the free energies of the phases 1, 2, ... that are in equilibrium, and $m_1, m_2, \dots, n_1, n_2, \dots$ are the number of moles of the components A, B, respectively, in each given phase.

From the thermodynamic condition of equilibrium (1) follow a number of elementary but important and interesting conclusions which we shall examine for the case of binary systems.

The free energy ΔF of alloy at a constant temperature and pressure P is equal to

$$\Delta F=\Delta H-T\Delta S=\Delta H_0+\Delta H_T+T(\Delta S_0-\Delta S_T) \tag{2}$$

(For condensed systems it may be assumed as an approximation that the Gibby free energy is equal to the Helmholtz free energy.) The change in the enthalpy ΔH and the entropy ΔS of the system may be resolved into two parts: ΔH_T and ΔS_T dependent upon the temperature, and ΔH_0 and ΔS_0 independent of the temperature. The temperature-dependent parts are small, and when the heat capacity is additive, are equal to zero for any alloys of the systems. The entropy of mixture of the solid solution characterizes the degree of disorder and may be computed on the basis of the well-known Boltzmann formula $S=k\ln W$, where k is the Boltzmann constant, W is the probability of the state.

For the case of a binary solid solution of a component A and a chemical compound A_pB in one another, one may write the following approximate expression for the entropy of mixture of one gram atom of the alloy:

$$S=R\ln W=-R\left\{(c+\delta)\ln(c+\delta)+(1-c-\delta)\ln(1-c-\delta)-\right.$$
$$\left.-\frac{p}{p+q}\ln\frac{p}{p+q}+\delta\ln\delta+\left(\frac{p}{p+q}-\delta\right)\ln\left(\frac{p}{p+q}-\delta\right)\right\} \tag{3}$$

where c is the molar concentration of the chemical compound in the alloy, δ = const. c is the concentration in the alloy of the atoms that are not occupying their places in the lattice of the compound (δ, consequently, characterizes the decrease in the

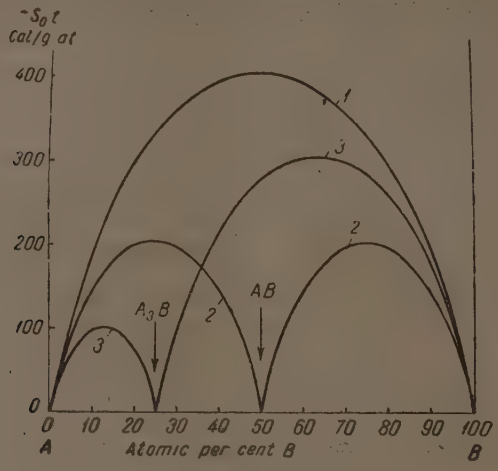

Fig. 1

degree of order in the alloy), and R is the gas constant. In Fig. 1 there is shown the change in the bound energy of mixture TS_0 at a temperature of 20° as a function of the concentration in a binary system for the cases of the formation of a continuous series of solid solutions between the components (curve *1*), between the components and the chemical compound AB (curve *2*) and between the components and the chemical compound A_3B (curve *3*). From the given graph it is seen that the maximum of the bound energy corresponds to a continuous series of solutions from A to B with a statistical distribution of atoms. The zero entropy of mixture for a completely ordered state is equal to zero.

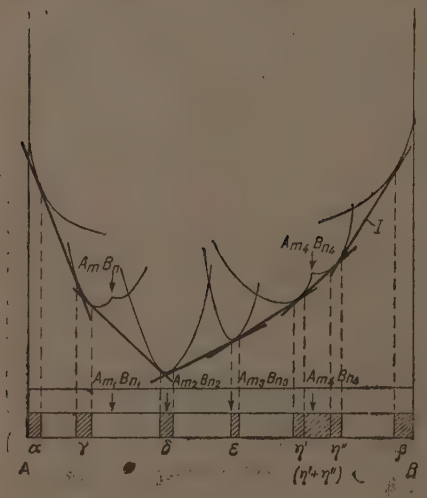

Fig. 2

From the condition of equilibrium (1) it follows that the curves representing the change in free energy as a function of the composition of the phases that are in equilibrium have a common tangent. Hence, the resulting (equilibrium) broken-curve I (Fig. 2) of the free energy as a function of the composition in a binary system, consists of linear segments corresponding to heterogeneous regions and of curved segments corresponding to the solid solution. From the condition of equilibrium (1) it follows, further, that the resulting curve I both as a whole and each curved segment of it is concave toward the minimum of the free energy for all phases of the binary system at given temperature. On these grounds it may be affirmed that in curve I there exists only one point corresponding to the minimum of the free energy in the system, except for the sole case when curve I is bound below by a straight line passing parallel to the axis of concentrations.

The curve representing the change in the thermal effect of the formation of the alloy from the components as a function of the composition will in its basic features follow the resulting curve of the free energy. However, on the curve of the thermal effects there may be minimums, maximums and points of inflexion. But the deviation of the curve of the thermal effects from curve *I* is equal in magnitude to the bound energy, *i. e.*, in the absence of intermediate phases, to not more than 0.4 of a kilogram calory per gram atom at normal temperatures and to less than that when intermediate phases are present.

When the heat capacity is additive, the free energy of the completely ordered state will be equal in absolute value to its thermal effect. Consequently, the smallest deviation will be in the region of the ordered state, the magnitude of which will be determined by the presence of a certain disorder and by a deviation from additivity of the heat capacities. That is why the course of the curve of the thermal effects as a function of the composition is simple, following in its basic features the course of the curve of the free energy, and why it is slightly dependent upon the presence of intermediate phases in the system.

The condition that the resulting curve of the free energy be concave towards smaller values of the latter leads, further, to the conclusion that a positive value of the thermal effect in the case of a binary continuously solid solution cannot be higher in absolute value than the bound energy, *i. e.* at room temperatures not more than 0.4 of a kilogram calory per gram atom, otherwise a break in solubility occurs.

The thermal effect of formation of a solid solution, both of the components in one another and of the solid solution of a component in an intermetallic compound may be conceived as consisting of two parts: a negative part conditioned by affinity, and a positive part conditioned by the difference in the atomic volumes of the components, by the repulsion of like atoms, by the heat of disordering, *etc.* In what follows, it is more convenient to represent the thermal effect in the form of the sum of two items:

$$\Delta H_0 = -\Delta H_c \pm Q = -\Delta H_c \pm [-\Delta H_{g1} + \Delta H_{g2}] \qquad (4)$$

where $-\Delta H_c$ is the negative part of the thermal effect, conditioned by chemical interaction, by the existence of a close order in the solution, by the presence of «complexes» of atoms of the components, corresponding in the nature and energy of the chemical bonds in them to an intermetallic compound of a definite stoicheiometric composition. This member is an additive function of the number of similar complexes and a linear function of the concentration in the case of a solution of the component in the intermetallic compound. $-\Delta H_{g1}$ is an additional negative part of the thermal effect, conditioned by the presence of auxiliary valence bonds in the components, by the chemical interaction of the complexes with the surrounding medium, by the tendency of atoms of one kind to surround themselves with as large as possible a number of atoms of the other component and, thus, satiate or utilize all possible chemical bonds, *etc.* ΔH_{g2} is an additional positive part of the thermal effect, conditioned by the presence of a distortion upon substituting an atom of one of the components for an atom of the other, or upon introducing an atom of one of the components into the lattice of the other, by the presence of forces of repulsion between like atoms, *etc.* In this member there should also be included the energy expended in the creation of a certain state of disorder (heat of dissociation). $Q\ [-\Delta H_{g1} + \Delta H_{g2}]$ is a minimum for the pure compound and component, reaching a maximum at certain concentrations of the solid solution, although, generally speaking, it may change its sign upon passing from the component to the compound. Obviously, the following three cases are possible upon the solution of one of the components in the chemical compound:

$$Q > 0, \qquad Q = 0, \qquad Q < 0 \qquad (5)$$

Upon the solution of both components in the chemical compound nine paired combinations are possible. These cases are shown graphically in Fig. 3.

Condition (5) on the strength of the unity of the structure of the chemical diagram (1) corresponds to the condition of possible formation of singular and non-singular maximums of various properties on the chemical diagram, and, in the first place, of the heats of formation from simple bodies. When $Q<0$, the maximum can

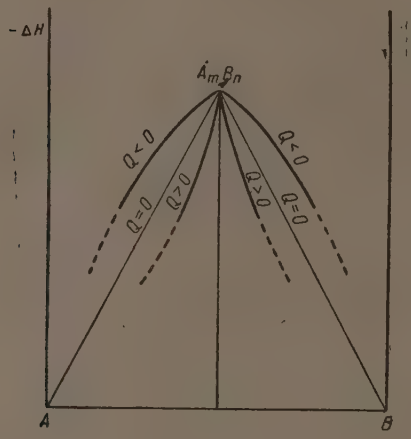

Fig. 3

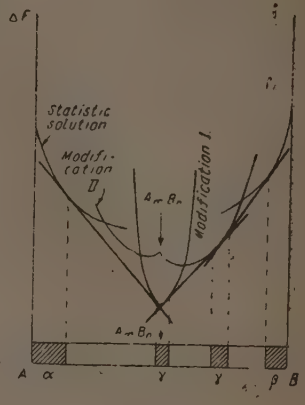

Fig. 4

be non-singular on both sides of the ordinate of the chemical compound, *i. e.* formation of a bertholletide will occur; when $Q>0$, the maximum will be singular, *i. e.* the formation of a daltonide phase will take place.

Obviously, the radius of curvature of the curve of free energy of each given phase as a function of the composition in the vicinity of the minimum will be determined by the relation between the absolute value of the thermal effect and the bound energy at a given temperature and by the relation between ΔH_c and Q. At small thermal effects and negative Q, the radius of curvature will be large and the minimums will lie on both sides of the ordinate of the chemical compound. On the contrary, at large heat effects, small values of the bound energy and positive Q, a singular minimum may occur on the curve of the free energy with reference to the ordinates of the chemical compound.

The enumerated conditions permit us, further, to elucidate the conditions of formation of the phases (2) representing a solid solution in the chemical compound which does not exist independently in conditions of equilibrium. The simplest cases of formation of such phases are shown in Fig. 2. From the given diagram it clearly follows that in a binary system under the indicated conditions both components may dissolve in themselves simultaneously only one chemical compound of the daltonide type, the singular minimum of the free energy of which corresponds to the minimum on the resulting curve I. We shall call this compound the central compound (3).

The phases of the daltonide type lying to the left of it are capable of dissolving only component A, and those lying to the right of it, only component B. What is more, the ordinate of the chemical compound may lie within the limits of the phase, on its boundary or, considerably rarer, outside the phase but near its boundary.

If the radius of curvature of the free energies of these phases is sufficiently great and a noticeable difference exists in the magnitude of the free energies at the minimums, these phases are sharply pronounced regions of solid solutions, in which orderliness will grow in the direction of the central phase. At small radii of curvature the points of contact of the tangents to the curves of the free energies of these phases will lie close to their minimums, the region of the solutions will be small.

For phases of the bertholletide type characteristic is the presence of a minimum on the curve of the free energy, away from the ordinate of the chemical compound. Therefore, most of the bertholletide phases on diagrams of state appear as individuals representing solid solutions in the chemical compound, the ordinate of which lies outside the region of the existence of the phase, which is shown schematically in Fig. 2 (phases γ).

Characteristic for bertholletides is the presence of a certain maximum on the curve of the free energy in the neighbourhood of the ordinate of the chemical compound, which follows from the expression for the entropy of mixture (3) and expression (5). That is why there is the possibility of a bertholletide phase forming, within which, in the region of the ordinate of the compound, there exists a break in solubility (phases η' and η'' in Fig. 2).

Of interest is the following particular case. At one and the same composition the formation of a compound in two different structural forms is conceivable. Under conditions of equilibrium there exists a compound with a structure corresponding to a smaller free energy. However, if the radius of curvature of the free energy of a non-realizable structure of the bertholletide type is considerably greater than in the existing forms of the daltonide type, then the conditions diagramatically shown in Fig. 4 are possible, under which there arises in the system an intermediate phase with an atomic structure corresponding to a non-existent modification of the compound.

Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 8

GEOLOGY

CHANGES IN THE RELIEF OF THE EARTH AS RELATED TO ITS MOVEMENTS AND ENERGY BALANCE

(CONTRIBUTION TO A CHARACTERISTIC OF GEOLOGICAL SPACE)

By B. L. LICHKOV

(Communicated by W. I. Vernadsky, Member of the Academy, 14.IX.1943)

The author's theoretical conceptions are based on the fact that the stability of this planet is determined by gravitational or molar forces which for aggregates of such a size as our globe, or even for mountain ranges and other large elements of the relief (plateaux, highlands, plains) by far exceed the molecular forces or forces of cohesion of the rocks composing these elements of the relief.

Hence it is the molar forces to which one should have recourse in explaining the statics and the dynamics of the Earth's relief and the cause of the main types of movements, the epeirogenetic and orogenetic one, which are distinctly different both in their nature and in time.

Two premises have been laid by the author at the basis of his reasoning when considering the causes of these two types of movements in the Earth's crust: 1) the differentiation between the nature of orogenetic and epeirogenetic movements and their extension in time as suggested in his previous work, and 2) the assumption that the theory of the Earth's contraction is fallacious, the Earth being not a body which undergoes cooling as a whole. Although the incongruity of the contraction theory for geological argumentation had long been made evident, no new universal theory was still developed which would comprise the two fundamental types of movement of the Earth's crust. As a core of such a theory the theory of isostasy may be regarded, which lays the maximum stress upon the rôle of molar forces in the life of the Earth's crust. It is, however, no consistent theory of movements, although such a theory could well be developed on the basis of the correct premises of the theory of isostasy. To do so it should be borne in mind that the mutual relations between the two types of movements, the orogenetic and the epeirogenetic one, which consist in the subordination of the two types to gravitational forces, are manifest in nature as the occurrence under the Earth's crust of subcrustal masses, a kind of «substratum» or base of the crust. It is the movements of the crust with respect to its substratum according to its consistency that actually underlie the changes of the relief—a fact which has not yet been fully realized by modern geologists. The decisive influence of the subcrustal substratum, of its states and movements, upon the relief of the Earth's crust may be accounted for by the alternating coolings and heatings of the superficial portions of the substratum, owing to which it by turns melts and solidifies. In the former case its consistency tends towards the liquid state, and then the Earth's crust hydrostatically floats in the substratum horizontally; in the latter case the consistency of the substratum

tends to the solid state, and then blocks of the crust rise to the surface, as a result of which vertical displacements take place. These changes in the main follow the scheme of Joly (1).

Let us begin with folds representing tangential contractions and crumplings of the Earth's crust during definite phases in the life of the planet. There is no room for doubt concerning their considerable magnitude, reaching—to judge from the magnitude of the folding now observed in mountain ranges—many hundreds of kilometres, which evidently corresponds to the dimension of slidings of the Earth's crust along the substratum. What is the cause of these slidings? They cannot be attributed to thermal contraction of the Earth: the ranges of these folds and of the thermal phenomena are incommensurable. In this juncture it should be admitted that these slidings in the Earth's crust may solely be due to collisions between blocks of the Earth's crust continents floating on the substratum, and to their mutual pressure, arising from the different inertia and different rates of movement. In other words, with the modern ideas of the tangential movements and the forces producing them, folding should be put down solely to the movement of continents which in the course of the history of the Earth used to bring them to collision and crumpling, as well as to a contraction of their borders. This was the idea conceived by E. Argand (2). The trends of these movements were dependent on the movements of the planet as a whole, in the first place, on its rotation (3).

There are two kinds of continuous movements of continents: their meridional progressive movement, and their western movement—lagging behind.

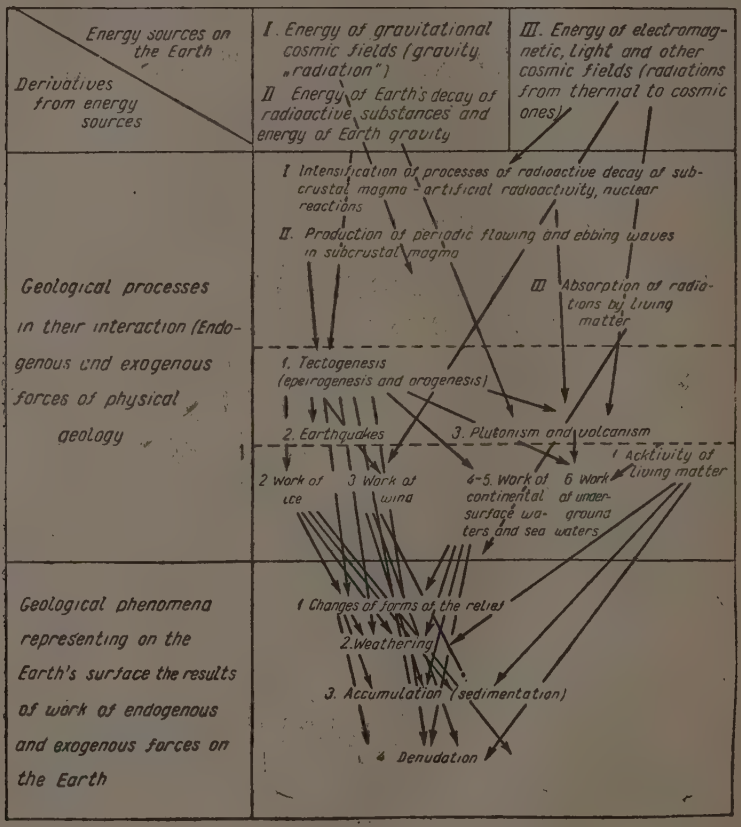
The latter has been long since associated by A. Wegener with the rotation of the Earth. The mechanism which is at work here may be understood as follows. Owing to a certain independence of deplacement with respect to the substratum in which they are embedded, in their eastward movement the continents are somewhat lagging behind to the west, the rate of this lagging being not the same for the continents because of their different mass and inertia. Etvesh and recently Wl. Köppen as well associated with the rotation of the planet also the meridional movements of continents. The main part belongs here to the inclines of the different surfaces of levels of the planet with respect to one another, and to the resulting shifts of the centre of gravity of the continental blocks with respect to the centres of gravity of the substratum displaced by them. Owing to this, a longitudinally directed item is produced, as confirmed by Epstein, Schweider and Lambert.

As to the vertical epeirogenetic movements, the case is similar. It should be borne in mind that the movements of the subcrustal substratum without which epeirogenesis cannot possibly be thought of—represent movements of two kinds, flowing and ebbing; the flows in subcrustal magma produce general elevations of the relief of the Earth's crust; the ebbs cause a levelling of its relief.

If the cause of the subsidences of the relief of certain areas of the Earth's crust may be logically—and in accordance with observation—sought for in the ebb of the magma produced, on the one hand, in connexion with the rise of the adjacent areas to which the magma is supplied, and on the other hand, with a sinking of depressions (to a very considerable degree under the weight of great terrigenous masses of rocks), for upheavals this explanation is untenable. Some elevations, it is true, involve a certain element of compensation with respect to the sinking taking place nearby. It stands to reason, however, that this explanation is insufficient, since it fails to account for the vast range of these elevations both in area and in height. Elevations play the chief part in mountain-building and should certainly have an independent cause.

It cannot be othervise, because the terrigenous loads causing the sinking of depressions—geosynclinal and terrigenous-geosynclinal—are a result of denudation which in order to exist requires a pre-existence of heights. The appearance of the latter should thus precede the sinking of depressions. The main flow-tide elevations of the substratum producing a new relief should therefore be due to a combined effect of the Earth gravity and that of other heavenly bodies—the Sun, the Moon, *etc.*, rather than to acceleration of the Earth gravitation. Consequently, they are

a manifestation of a peculiar resonance (R. Schwinner) similar to those which are displayed in the tide phenomena of the earthly ocean. It is only the form of manifestation of the resonance in the subcrustal magmas that is different—in connexion with the periodic changes of their consistency, as well as with the fact that the tide phenomena here are not directly observed, but displayed in the changes of the relief of the solid crust. The well-pronounced rhythm of these movements is quite peculiar. The epeirogenesis is thus also due to a cause lying outside the planet.



Our final conclusion as regards the movements of the Earth's crust is as follows. We are unable to gain a clear idea of the causes underlying either of the two kinds of movements without relating this movement in the Earth's crust with the movement of the planet as a whole, and through it with the sources of energy outside the Earth, *i. e.* through the cosmic spaces with other heavenly bodies. The energy balance of the field is, as it were, supplemented here, using the expression of A. Einstein, by quantum phenomena of the transmission of states, which results in a kind of force- and wave-fields. In the field resulting, enveloping the planet from outside, there are, using Einstein's words again, two «logically incompatible parts: aside from gravitational forces, also electric and magnetic forces». As has been stressed by W. I. Vernadsky, this is manifest in the fact that the surface of the planet receives a vast amount of radiation in the form of waves, ranging over more than forty octaves: from decimal fractions of a millimeter to one kilometer long. These

waves transmit the energy states from the cosmic medium to the planet, this energy being absorbed partly by the living matter of the planet; both ways thus lead to the production and maintenance of geological processes.

What has been said here may be summed up in the following way. The formation on this planet of a new relief consists in two kinds of space entering into contact at the boundary of the surface of the planet within the field enveloping the Earth. Of these two kinds of space that of the planet, characterized by a condensation of matter, is the space of the field of gravity, with the fields of cohesion subordinate to it, where the force is determined by distance and mass. A. Einstein, entirely adopting from Faraday and his followers (Maxwell and others) the conception of force fields and having thus discarded Newton's «action at a distance», which had perplexed more than one by its obscurity, decisively advanced the problem of the existence of different kinds of space. Recently W. I. Vernadsky made the geologists face the problem of geological time, by having stressed the existence of different kinds of time. Geological phenomena that take place on the surface of the planet or in its proximity, undoubtedly suggest a problem analogous to that of the geological time—the problem of geological space.

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Received
14.IX. 1943.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
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MINERALOGY

RESIDUAL CLAY FROM DOLOMITIC FLOUR FROM THE UPPER CARBONIFEROUS DEPOSITS OF THE SAMARA BEND

By F. P. SAVARENSKY, Member of the Academy

The recent works carried out by N. V. Frolova, E. S. Salmanson, N. V. Soloviev of the Institute of Geological Sciences of the Academy of Sciences have fully confirmed the previous views concerning the origin of the so-called «dolomitic flour» ([1–3]). Vast deposits of dolomitic flour were encountered in the course of reconnoitry work for the construction of the Kuibyshev water-power plant among the limestones and dolomites of the Upper Carboniferous and Permian in the Krasnaya Glinka and Perevoloki region. The dolomitic flour is a product of the leaching-out and decomposition of dolomitic limestones and dolomites, which has remained in place amidst the rock still preserved.

Rocks of the dolomitic flour type occur not only at the Samara Bend, but also in the dolomitized limestones of the Upper Carboniferous of the Oka-Tzna uplift, *e. g.* at Kovrov, near Shchekino in the vicinity of Moscow; small inclusions were found in limestones when carrying out investigations at the site of the Palace of Soviets. Farinaceous dolomites were recorded also by G. Theodorovich ([4]) in the vicinity of Dobryatino, Moscow-Kursk railway; this writer observed among farinaceous friable dolomitic rocks the presence of yellow-brown and greyish-brown clays as a residual material left after the leaching of dolomitic rocks. N. V. Frolova recorded dolomitic flour in the rocks of the Middle Cambrian of the Lena-Baikal divide.

Observations have shown that between the friable dolomitic flour and the hard dolomitic limestone there exist all the intermediate varieties.

The analyses of the dolomitic limestones and dolomites of the Samara Bend point to the presence of small quantities of other minerals in addition to calcite and dolomite. The same is observed also in the dolomitic flour, the insoluble residue of which constitutes from fractions of a per cent to several per cent.

Amidst the dolomitic limestones passing into flour the residual clays occur, however, as small accumulations within a dolomitic flour deposit. In the region of the Kuibyshev water-power plant such occurrences have been encountered. From their external appearance they are white or ochre-yellow argillaceous rocks which are leaner in the white varieties and fat to the touch in the ochreous portions. The external appearance of this rock would make one think that its white portions contain calcite or dolomite. The ochrous substance in the piece of rock examined stained its lower part and spread not quite uniformly, but imparting to it a watery pattern.

When dry this clay has a rather high density and shows no porosity perceptible by the naked eye.

Two pieces of this residual clay were studying more thoroughly: one (I) from the upper portion of the piece, a white leaner clay, and another (II) from the lower part, an ochrous yellow fat clay.

Table 1

Results of Granulometric Analysis
(per cent of particles)

Specimen	Specific gravity	Diameter of particles in millimeters					
		0.5—0.25	0.25—0.05	0.05—0.01	0.01—0.005	0.005—0.001	less than 0.001
I	2.74	—	0.21	67.17	7.16	22.03	3.43
II	2.75	0.04	1.68	72.38	6.74	18.26	0.90

The largest number of particles falls to the share of the fraction of particles from 0.05 to 0.01 mm in size, which is probably accounted for by the considerable content in the rock of dolomitic flour, in which particles from 0.25 to 0.01 mm are obviously prevailing.

A chemical analysis of these two specimens gave the following results (Table 2).

Table 2

Content in per cent	Specimen I	Specimen II
SiO_2	26.04	46.99
Al_2O_3	13.08	22.27
Fe_2O_3	1.10	3.10
FeO	none	traces
TiO_2	0.15	0.39
MnO	none	none
CaO	15.43	2.47
MgO	11.99	4.41
S	none	none
SO_3	»	»
H_2O	2.10	7.13
Loss on ignition	27.16	10.33
Total	97.05	97.09

If we consider that the alkaline earths enter chiefly into the composition of carbonates, which is in agreement also with the high value of the loss on ignition, we obtain 52 per cent carbonates in specimen I, and over 13 per cent in specimen II, while the content of quartz and of the aluminum silicate portion increases from 39 per cent in specimen I to 67 per cent in specimen II. Thus, these two specimens may be regarded as two transition stages from dolomite to clay.

The clayey fraction of these specimens was subjected to an X-ray study in the Soil Institute of the Academy of Sciences. I. D. Sedletzky gave the following determination of the minerals composing the clayey portion: montmorillonite, kaolinite, and, in addition, dolomite and quartz. The thermal analysis made by L. G. Berg (Institute of General and Inorganic Chemistry) revealed in the main the presence of montmorillonite.

The origin of montmorillonite in the residual clay of weathering dolomites still remains obscure. Neither it is clear whether it had already taken part in the material of the sediments of the sea basin as montmorillonite, or its origin was connected with the zone of weathering. In order to settle this question further information is necessary.

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Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 8

PALAEONTOLOGY

A REPRESENTATIVE OF A NEW GROUP OF INSECTS FROM THE PERMIAN DEPOSITS OF THE URALS

By G. M. ZALESSKY

(Communicated by A. A. Borissiak, Member of the Academy, 23.I.1944)

The remain of the wing of a Permian fossil insect from the Bardian-stage deposits in the basin of the Sylva river, Urals, a description of which is given below, is so peculiar that it cannot be referred to any of the orders known. The venation

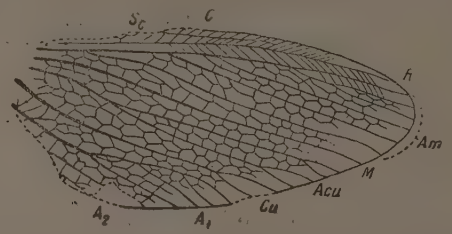

Fig. 1. Wing of *Bardapteron ovale* nov. gen. et nov. sp.

of this wing somewhat reminds that of the form *Eohymen maculipennis* Martynov, referred by Martynov (1) to *Protohymenoptera*, but this likeness is so slight that it does not justify a correlation of the two forms. The resemblance lies, first, in the nature of the anal vein and the presence of cross-veins between R and Am, running obliquely with an incline to the left, while similar cross-veins in the subcostal field are directed to the right; and secondly, in a certain simplicity of the main veins which are otherwise different. In Am being independent of R, the form described approaches to the mayflies to which this character is peculiar. Another feature in common with mayflies and dragonflies is the network of veins. The form here described is separated out into a new order and a new family for the time being typified by a single genus.

Order *PERMODICTYOPTERA* nov. ordo
Family *BARDAPTERIDAE* nov. fam.
Genus *Bardapteron* nov. gen.

Wing large, ovate, with a very delicate subcosta. R simple. Am independent of R throughout its length; it might touch R at the very base. Am branching, giving off a number of branches backwards. M simple. Approaching the middle of the wing Acu gives off two branches, dichotomizing at the ends. Cu simple over the greater part of its length and dichotomizing at the end only. A_1 ramifying into four branches. A coarsely-cellular network of veins all over the wing. In the costal and subcostal fields and between R and Am regular cross-veins are arranged at an angle to

the main veins, in the costal and subcostal fields with an incline towards the apex of the wing, and between R and Am, with an incline towards its base.

Bardapteron ovale nov. gen. et nov. sp.
(Figs. 1 and 2)

Imprint of a large wing, 40 mm long, in the median part 18 mm wide. The integer wing was probably about 42 mm long. The very base of the wing is not visible, being concealed under a piece of hard rock which it is impossible to remove.

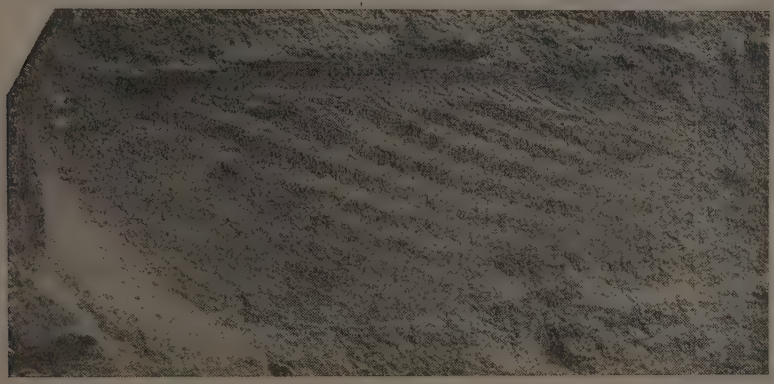

Fig. 2. Photograph showing the imprint of *Bardapteron ovale* nov. gen. et nov. sp.

The shape of the wing is irregularly ovate. This seems to be a posterior wing, for the anal portion of it is rather wide, and the posterior edge of the wing is sufficiently convex. The anterior edge of the wing has not been fully preserved, C is preserved only in the distal part. Only the distal part of the subcosta (Sc) has been preserved, being represented by a very slender, delicate vein. R simple. In the distal part of the wing the cross-veins run forwards of R, *i. e.* between Sc, the edge of the wing and R, are arranged obliquely with an incline to the right (as viewed on the imprint), that is to the apex of the wing, while similar cross-veins between R and Am are arranged obliquely with an incline to the left, that is towards the base of the wing. This gives it a rather peculiar appearance, the radius looking like a lufty feather or like an ear. A number of similar veins is observed also between the anterior branches of Am. From the type of branching Am belongs rather to the pectinate type; one of the four branches given off almost parallel, dichotomizes. Further backward there is a simple vein which seems to correspond to M. It is followed by a vein which somewhat before reaching half its length dichotomizes into two branches; these dichotomize once more at their ends. This is probably Acu. The next vein dichotomizing only at its end, is Cu. Still more posteriorly, there is located A_1 giving off branches towards the posterior edge of the wing; four of them are preserved on the imprint. Parts of A_2 are perceptible. The basal part has not been preserved, which makes it impossible to ascertain whether there was another anal vein closer to the base of the wing. Between the main longitudinal veins there is located a coarsely cellular network of cross-veins. In some cases the cells of this network are rather regularly pentagonal, forming at the places of contact a zigzag line, something like zigzag inserted veins. In the imprint the zigzag lines are located in depressions, while the rest of the veins are convex, except for R and the main stem of Am which are also in depressions. The imprint is obviously a negative one, that of a left wing.

Locality: left shore of the Barda river, Krutaya Katushka baring. Collected by H. T. Mauer, 1933.

The taxonomic position of the new form of *Bardapterón ovale* nov. gen. et nov. sp. remains as yet uncertain; it is evident, however, that it should be considered as a separate group, most probably a new order, to which the name of *Permodictyoptera* has been given above. The *Permodictyoptera* seem to descend from the same ancestry as mayflies and to approach closely the stem of such *Palaeoptera* as *Plectoptera*, *Odonata*, and *Protohymenoptera*.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 8

GENETICS

GENETIC DIVERSITY OF GAMETES IN THE FLOWER OF COTTON-PLANT

By D. V. TER-AVANESJAN

(Communicated by B. A. Keller, Member of the Academy, 21.I.1944)

The influence of external, ecological conditions upon the vegetable organism has been proved by numerous facts of observation and experiment. During the ontogenesis the plant is more or less affected by the environmental conditions; owing to this, even a so-called pure-line variety does not contain two plants identical as to their qualitative and quantitative characters.

The external conditions affect not only the organism as a whole, but separate organs of the plant, as well.

By assuming that the external conditions affect all the elements of the plant we call in question the qualitative homogeneity of the pollen (male gametes) in the flower, supposing that the pollen grains of the flower must also be affected by them.

In order to prove this assumption we undertook experiments involving pollination of the cotton-plant flower with a limited amount of pollen; the possibility of selection of such pollen grains as were the most adapted to the given conditions thus being eliminated, pollination with random pollen should necessarily take place.

Such a mode of pollination was likely to disturb the equilibrium within the variety, and to bring about intravarietal segregation in the progeny.

The experiment was conducted at the Central Asia Department of the All-Union Institute for Plant Breeding near Tashkent, in 1940. As an object for the work, we chose the cotton-plant variety C-15 Upland (*Gossypium hirsutum*) and 35-1 (*G. barbadense*), from Egypt.

At the evening preceding the experiment the flower was emasculated, the green stamens put into a small packet which was fixed to the pedicel and left so until the next day. On the next day the ripe pollen was poured out of the packet on to an object-glass, a strictly definite number of the pollen grains were counted under a magnifying glass and transferred with the aid of a minute brush upon the mature stigma.

It should be noted here that the ovary of a cotton-plant contains up to 40 egg-cells, and that there is about 10000 pollen grains in a flower.

In our experiment we transferred to the stigma up to 20 pollen grains, and the number of seeds formed was found to vary between 8 and 15 per boll.

The plants obtained from the experimental seeds and sown in 1941 showed a development different from that of the controls pollinated in the usual way; the difference could be observed from the very moment of appearance of the first four leaves. In the subsequent the difference became gradually more and more conspicuous both in the American and Egyptian cotton-plants; as a result, by the end of the growing period we had instead of the pure elite varieties a variegated population, while the control plants showed a uniform mode of development.

An analysis of the qualitative and quantitative characters of each experimental plant taken separately went to confirm the existence of a population in the progeny of a plant pollinated by a limited portion of pollen.

Let us illustrate the above-said by giving here a few figures from the analysis of individual F_1 plants. In the variety C-15 the yield of lint per shrub varied among the control plants within a range of 180—220 g; in the experimental plants this character was subject to still more conspicuous variations, ranging from 82 to 510 g per plant. The length of the fibre also differed greatly: in control plants it varied between 31.0 and 31.4 mm; among the plants of the population produced by us, from 26.4 to 35 mm. Sharply deviating figures have been obtained for the rest of characters both in variety C-15 and among plants of the Egyptian variety 35-1.

In 1942 we sowed the seeds collected from each plant by individual families in order to check up in the F_2 the properties acquired in the F_1; there were up to 60 experimental plants in each row, and as many controls. The experimental plants within a single line proved identical as to their morphological characters. The analysis of the characters in the plants of each separate row revealed a most curious picture.

The range of variation of the separate characters is made quite evident by the figures summarized in Tables 1 and 2 for the varieties C-15 (Upland) and 35-1 (Egyptian).

Table 1

Variations in Characters among Families in F_2 of the Variety C-15 (Upland)

Number of bolls per plant (average)		Total yield per plant in g (average)		Length of fibre in mm		Output of fibre in %		Height of plant in cm	
contr.	exper.	contr.	exper.	contr.	exper.	contr.	exper.	contr.	exper.
25.0	21.3	138.5	115.0	32.1	31.7	32.2	31.7	72.8	67.6
25.3	23.9	144.4	128.0	32.1	32.8	32.4	32.7	73.9	69.4
25.9	24.3	148.9	131.2	33.1	33.7	32.8	33.2	74.5	71.3
25.9	25.8	149.0	148.2	33.6	34.2	33.0	34.2	78.0	73.0
	26.7		159.2		34.9		38.2		75.5
	27.2		167.6		36.3				76.5
	28.9		173.3		36.7				77.9
	32.7		192.3		36.8				81.0
	34.5								88.5

As may be seen from the tables, we have obtained for all characters a transgression which in its right part is of great practical interest.

The best lines of the F_2, both for the C-15 and 35-1 varieties, were sown in 1943, and at the time being, so far as it is possible to judge from the preliminary data, they are much superior to the control plants.

Thus, from the abundant experimental evidence accumulated in the course of the last years both for the varieties listed above and for several others which were used for checking up the experiments, we arrived at the following conclusions:

1. Under natural conditions the flower is, as a rule, fertilized with biologically incompatible pollen, *i. e.* such a pollen as, being adapted to the environmental conditions, reproduces the type of the plant (variety).

2. By fertilizing a flower with a limited number of pollen grains (biologically incompatible) we obtain a progeny consisting both of plants much inferior to the controls, and excelling them in all characters.

3. The obtaining in the F_1 of populations by pollinating the stigma of a flower with 20 pollen grains is evidence of the qualitative heterogeneity of the male gametes within a single flower and of their great potentiality.

4. Assuming that, as said above, the pollen may be qualitatively not uniform, we suppose the population arising in the F_1 to be due to intrafloral hybridization.

Table 2

Variations in Characters among Families in F_2 of the Variety 35-1 (Egyptian)

Number of bolls per plant (average)		Total yield per plant in g (average)		Length of fibre in mm		Output of fibre in %		Height of plant in cm	
contr.	exper.	contr.	exper.	contr.	exper.	contr.	exper.	contr.	exper.
47.7	41.7	161.9	137.6	34.1	29.6	32.1	29.7	139.6	138.0
48.7	44.9	162.2	142.1	34.4	30.3	33.9	31.5	143.3	142.4
49.1	46.0	165.5	161.0	34.8	32.8	34.0	32.7	143.4	144.3
49.8	50.8	169.5	164.8	34.9	33.6	34.1	33.8	144.7	146.1
53.4	53.1	175.1	175.1	35.1	34.0	34.3	34.6	147.9	147.5
53.6	55.6	176.3	177.1	35.5	34.2	34.3	35.1	149.0	149.7
54.2	60.0	184.2	179.2	35.7	34.9	34.5	37.3	152.0	150.4
	63.8		181.7	35.8	35.1	35.0	39.3		151.7
	64.7		192.0		35.7				—
	65.6		192.0		35.7				155.6
	65.6		216.9		35.8				157.9
	70.0		231.0		36.1				166.5

5. The fact that no segregation takes place in the F_2 is accounted for by intrafloral hybridization being essentially different from the intervarietal one.

6. The method of limited pollination here described may be applied to practical breeding, in the cases where it is necessary in the course of 2—3 years to improve an elite varietal material, or to obtain a new variety from a variety, or even from an individual plant within a very short period of time.

7. The observational data pointing to the genetical diversity of the pollen grains within a flower are of considerable theoretical interest and may be turned to account for a number of biological questions.

The experiments here described are in progress.

Received
4. II. 1944.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 8

PLANT PHYSIOLOGY

CONTRIBUTION TO THE ANALYSIS OF THE THEORY OF FLOWERING OF PLANTS

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(Communicated by A. N. Bach, Member of the Academy, 2.IX.1943)

In his attempts to inquire into the physiological nature of the processes which make the plants ready for flowering, Klebs (4) arrived at a conclusion that the internal condition responsible for the plant's breaking into blossom consists in the accumulation in its tissues of carbohydrates and in a reduced content of nitrogen compounds.

Klebs' theory of flowering enjoyed universal recognition during about 20 years; it makes part of any manual or textbook of plant physiology. By and by numerous facts were secured, however, which defied to be framed into this theory refuting it; foundation was laid to the hormonal theory of flowering, advanced by us in 1937 (1). Investigations carried out by Moshkov (3) went materially to support this theory, while works by Hamner and Bonner (5), Borthwick and Parker (6), Löwing and others confirmed the main facts it had been based upon.

The next step of the work was to isolate the hormonal substances of flowering into non-living medium and to make it act upon the plant from without. The numerous experiments carried out by myself within 1936—39 at the Institute of Plant Physiology of the Academy of Sciences of the USSR, as well as experiments made by Hamner and Bonner at the Chicago and California University, failed to give any positive results as yet. It became clear that one cannot possibly succeed in solving this problem without possessing at least a thorough knowledge of the fundamental properties of these substances, of the correlation between the formation of the hormonal substances of flowering and the rest of chemical substances known to us.

In 1940 we started experiments in this direction at the Institute of Plant Physiology of the Academy of Sciences of the USSR. Some of the tests led to quite definite results; the two most essential facts secured from experiments with *Perilla nankinensis* are described in this note.

E x p e r i m e n t 1. Carried out in 1940. Young perilla plants were transplanted on April 23 from boxes into earthern pots 4 inches high and grown until April 22 under conditions of a long natural day. On April 22 we selected 6 plants (in 6 pots), equal in size and of similar general appearance, and classed them into three groups: 1) two plants were left under conditions of natural day, 2) two others were transferred to continuous darkness, being covered with a veneer box in a dark photoperiodic greenhouse, at usual summer temperature; 3) the remaining two plants were transferred to continuous darkness at a temperature of 3—6° C in a cold cellar. The plants were kept in the darkness during a fortnight, after which they were transferred on July 6 to trucks and given short-day (10 hours) illumination (light was supplied to them from 8 a. m. till 6 p. m.). At the same time the plants of the first group were also transferred from long natural day to short-day conditions.

The purpose of the experiment was to evaluate the effect of a pre-treatment with darkness upon the rate of flowering of the plants. Out of the two plants which had

been kept in continuous darkness at relatively high temperatures, the one perished, another was found to have its main terminal bud and a part of its leaves injured. And yet it was this very plant that started budding the first (18.VII); next fol-

Fig. 1

lowed the plants of the set kept in the dark and at low temperatures (20.VII); the last to start budding (26.VII) were the control plants which had been transferred to short day directly from the long day. Corresponding shifts could also be noticed in the dates of flowering. Fig. 1 illustrates the state of the plants by August 31: the plant No. 1 is in the phase of budding, while those of the dark set kept in the cold (pot No. 3) and those of the dark set kept at high temperatures (pot No. 5) are covered with blossoms.

The experiment makes it quite clear that the plants which were kept during 14 days in the dark, being entirely exhausted and deprived of carbohydrates, are more susceptible to the effect of a short day and pass to reproductive development after some twelve or fourteen days. As a rule, perilla plants do not start budding until nineteen-twenty days after being transferred to short-day conditions, as may be seen in the control plants of the same experiment, as well as in many other experiments earlier described. A similar experiment carried out with *Perilla ocymoides* gave analogous results.

Experiment 2. Carried out in 1941. Young perilla plants were transplanted on June 4 into battery-glasses of a 0.7 kg capacity, into washed sand, with full Prianishnikov solution, save for ammonium nitrate, a quarter of the normal ration being supplied to eight glasses, and a double normal ration, to as many. One half of the plants of either group was left at a natural day-length, another was grown under conditions of a short, ten-hour day.

The object of this experiment was to bring out the effect of nitrogen food upon the rate of blossoming of the plants. The plants transferred to natural long day failed to flower. Those kept at a short day and given a double ration of nitrogen food started flowering by July 16, much sooner than the plants kept at a reduced nitrogen diet, which did not flower until 29. VII. The size and the dry weight of a plant given a double nitrogen ration considerably exceeded those of plants kept at a quarter of the normal ration (27 cm and 2.44 g against 7 cm and 0.21 g).

This experiment makes it evident that when liberally supplied with nitrogen compounds, the plants become more susceptible to the effect of short day, and break into blossom sooner than plants poor in nitrogen compounds.

If we are to compare the data of the two experiments, we soon become aware of the fact that the results are in both cases opposite to what has been known since the time of Klebs about the influence of nitrogen-carbohydrate exchange upon the flowering of plants.

These experiments served us as a basis for our further investigations carried out in 1942, at the Erevan State University, in the greenhouse of the Department of Plant Anatomy and Physiology, under collaboration of S. V. Melikjan and S. A. Asatrjan of that department. The experiments involved two short-day species: the same *Perilla nankinensis* again, and *Chrysanthemum indicum*.

Our main idea was to enrich in nitrogen compounds or carbohydrates not the plant as a whole, but directly those very leaves in which the formation of the hormonal flowering substances is known to proceed under favourable conditions of light regime.

With perilla we carried out three experiments, the pre-treatment of the plants and the treatment of leaves being carried out according to the same scheme, save for some insignificant deviations. We took for experiment mature plants of identical age, which had been kept continuously under conditions of natural day. The top of the plant, all its leaves and shoots were cut of, except for two opposite leaves with their axillary shoots. The left leaves of all the plants were kept under conditions of natural day and served as controls: the right leaves were kept under conditions of a short, 9-hour-long day. Moreover, the right leaves were immersed into 1 per cent solutions (in the 3rd experiment, into 0.5 per cent solutions) of the following substances: 1) ammonium nitrate NH_4NO_3, 2) potassium nitrate KNO_3 (in the 3rd experiment, of sodium nitrate $NaNO_3$), 3) asparagin and 4) glucose. Immersion lasted one hour, and two hours in the 3rd experiment.

In the observations stress was mainly laid upon the recording of the phase of budding and flowering of the right shoots which at the start of the experiment had been in the state of vegetation in all plants (2).

The dates of exposure of the leaves to short photoperiods, of their immersion into the solutions, and the results of the treatment are given in Table 1.

Table 1

Influence of Nitrogen Compounds and Glucose upon the Rate of Blossoming in Leaves Immersed into Solutions of These Substances

No. of exper.	Period of exposure to short day	Period of immersion	Number of immersions	Concentrations in %	Date of flowering					
					natural day	short day, 9 h.	Short (9-hour) day			
							NH_4NO_3	KNO_3 ($NaNO_3$ in 3rd exp.)	asparagin	glucose
1	14.VII—5.IX	20.VII—14.VIII	8	1	no	12.VIII	12.VIII	12.VIII	14.VIII	16.VIII
2	16.VII—5.IX	20.VII—14.VIII	8	1	no	15.VIII	17.VIII	—	18.VIII	22.VIII
3	17.VIII—19.IX	12.VIII—5.IX	10	0.5	no	12.IX	—	10.IX	10.IX	13.IX

Although, as it may be seen from the table, the dates of breaking into blossom do not vary greatly, nevertheless a lagging behind of the flowering in the case of enrichment of the leaves with glucose is conspicuous in all the three experiments. An increase in the content of nitrogen compounds in the leaves was found to accelerate the blossoming of the shoots in the 3rd experiment only, while in the two

others it set in either simultaneously with the controls (short-day leaves), or else by some 1—2 days later. This should be accounted for by the fact that when immersed the 1 per cent solutions, the leaves often became injured, so that by

Fig. 2

the beginning of flowering they had necrotic spots or dried-up areas along their margins. In the cases where 0.5 per cent solutions were made use of, this phenomenon was less pronounced if at all.

The experiments with chrysanthemum were carried out with adult plants of the variety «Rayonnante» grown in earthern pots. The experiments were started on July 22, and carried on according to the following scheme: 1) natural long day, 2) short day (9 hours), 3) short day and nitrogen supplied to the leaves, all the leaves of the main stalk being profusely wetted with a 1 per cent solution of KNO_3 at five dates: on 22.VII, 30.VII, 3.VIII, 12.VIII, and 7.IX; 4) short day and nitrogen supply to the soil; the soil of the pots was irrigated with 1 per cent solutions of nitrates, at a ratio of 1 g nitrates per 1 kg of soil; irrigation was done at four terms, *viz.* on 22.VII, 30.VII, 12.VIII, and 7.IX; 5) continuous darkness during 5 days (22.VII—27.VII) and short day (27.VII—5.IX); 6) darkness, short day and nitrogen supply to the soil.

The data secured from our experiments with chrysanthemum, which were achieved by October 10 are illustrated by Table 2.

Table 2

Growth and Development of *Chrysanthemum indicum* as Affected by Nitrogen Compounds and Darkness

Development of chrysanthemum	Long natural day	Short 9-hour day	Short day+ nitrogen supply to leaves	Short day+ nitrogen supply to soil	Darkness+ short day	Darkness+ short day+ nitrogen supply to soil
Date of flowering .	no	23.IX	14.IX	14.IX	23.IX	21.IX
Height of plant in cm	87	76	86	99	95	101
Number of blossoms per plant	0	9	14	27	11	31

From the table it may be seen that by enriching the plants in nitrogen compounds supplied either directly to the leaves, or by way of the roots, the flowering of *Chrysanthemum indicum* was made to set in at a much earlier date (*viz.* on September 14, whereas the short-day controls did so on September 23). When the plants were preliminarily kept in continuous darkness, the five-day period of darkness was found to speed up their flowering by as much or even more than the five-day period of short days.

The height of the plants, the profuseness of their branches and blossoms were increased both by enriching the plants with adequate amounts of nitrogen compounds (the effect is better when the latter are introduced into the soil than in case of wetting the leaves), and by previously keeping them in continuous darkness.

Fig. 2 represents three chrysanthemum plants of the variety «Rayonnante». The left-hand short-day plant is at the phase of budding, in the middle is a short-day plant, given potassium nitrate *per via* leaves at the phase of flowering; the right-hand short-day plant was supplied nitrates to the soil also at the phase of flowering.

The results obtained permit of the following two conclusions.

1. The flowering theory of Klebs has heretofore been subject to criticism based largely upon such analytical data as revealed the disharmony between the dates of the plant's breaking into blossom and the ratio carbohydrates/nitrogen compounds it requires; the experiments here described prove Klebs' theory to be incosistent on the basis of data obtained by the synthetic method.

2. The plant's flowering and the processes by which it is prepared are associated with phenomena of regulatory nature, and do not depend on the ratio of the nutrient substances. The formation of hormonal flowering substances in plants of several short-day types is connected, however, with the presence of a large amount of nitrogen compounds and a relatively low content of carbohydrates.

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Received
2.IX. 1943.

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